

U.G. 5th Semester Examination-2025**STATISTICS****[MAJOR-VI]****Course Code : ST-M-5.1-T****[Theoretical]****(Inference-II and Large Sample Theory)****[NEP-2020]**

Full Marks : 50(40+10)

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meaning.*

1. Answer any **five** questions: $2 \times 5 = 10$
- a) Define UMPU test.
 - b) Distinguish between level and size of a test.
 - c) State Slutsky's theorem.
 - d) Let $\{X_n\}$ be a sequence of independent random variables with $P[X_n = \pm\sqrt{n}] = \frac{1}{2}, \forall n$.

Does Chebyshev's WLLN hold for $\{X_n\}$?
Justify.
 - e) Define ASN in connection with SPRT.

- f) Let $\{X_n\}$ be a sequence of iid $N(\mu, \sigma^2)$ random variables. If $\bar{X}_n$ be the mean based on first n random variables and

$$s_n^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X}_n)^2,$$

show that s_n^2 converges in probability to σ^2 .

- g) What do you mean by power of a test?

- h) What is CLT?

2. Answer any two questions: $5 \times 2 = 10$

- a) Let $X_1, X_2, \dots$ be iid random variables with $E|X_1| < \infty$. Let N be a stopping random

variable and $S_N = \sum_{i=1}^N X_i$. If $E(N) < \infty$, show that $E(S_N) = E(X_1) E(N)$.

- b) Prove that convergence in probability implies convergence in law.

- c) Let $X_1, X_2, \dots, X_n$ be a random sample from a distribution with pdf.

$$f(x, \theta) = \theta x^{\theta-1}, 0 < x < 1.$$

Find the MP size α test for testing

$$H_0: \theta = 1 \text{ against } H_1: \theta = 2.$$

- d) Let p be proportion of success in n independent Bernoulli trials with success probability π . Show that as $n \rightarrow \infty$,

$$\sqrt{n}(\sin^{-1} \sqrt{p} - \sin^{-1} \sqrt{\pi}) \xrightarrow{L} N\left(0, \frac{1}{4}\right).$$

3. Answer any two questions: $10 \times 2 = 20$

- a) i) Suppose $(X_1, X_2, \dots, X_n)$ be the observations of a random sample of size n from a Cauchy distribution with pdf

$$f(x) = \frac{1}{\pi[1+(x-\theta)^2]}, -\infty < x < \infty, -\infty < \theta < \infty$$

Obtain the asymptotic distribution of the sample median for large n .

- ii) Let $X \sim B(n, 0.45)$. Use CLT to find n such that $P\left(X < \frac{n}{2}\right) \geq 0.90$. $6+4$

- b) i) Let $(X_1, X_2, \dots, X_n)$ be a random sample from $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown. Find the size α LR test for testing $H_0: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$.

- ii) Find the MP size α test for testing $H_0: X \sim N(0, 1)$ against $H_1: X \sim g$, where $g(x) = \frac{1}{2} \exp(-|x|), -\infty < x < \infty$. $6+4$

- c) Derive the SPRT of strength (α, β) for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1 (> \theta_0)$ in sampling from a normal density

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{x - \theta}{\sigma} \right)^2 \right], -\infty < x < \infty$$

where σ is known. Also obtain its OC function.

- d) i) Describe chi-square test of homogeneity.
 ii) The following information is given on the prevalence of a disease in two communities:

Community : A B

Population proportion : π_A π_B

Sample proportion : p_A p_B

Sample size : m n

Obtain a $100(1-\alpha)\%$ confidence interval for $\pi_A - \pi_B$ 5+5

[Internal Assessment:10]