

U.G. 1st Semester Examination - 2022**PHYSICS****[PROGRAMME]****Course Code : PHYS-G-CC-T-01(A, B & C)****Full Marks : 40****Time : $2\frac{1}{2}$ Hours***The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.***Answer all the questions from selected Option.****OPTION-A****PHYS-G-CC-T-01(A)****(Mechanics)****GROUP-A**

1. Answer any five questions: $2 \times 5 = 10$
- a) Show that areal velocity is constant for planetary motion.
 - b) Write down the necessary and sufficient condition of a first order differential equation to be exact.
 - c) State and prove the work energy theorem.
 - d) Define radius of gyration for a rigid body rotating about a specified axis.

[Turn over]

- e) What is Poisson's ratio of a rigid body?
- f) Determine the dimension of the coefficient of viscosity of a liquid.
- g) Show that the field intensity is perpendicular to the displacement vector in an equipotential surface.
- h) Show that under Galilean transformation, acceleration of a particle remains invariant.
- i) Define Coriolis force with a vector diagram.

GROUP-B

2. Answer any two questions: 5×2=10
- a) Find the equation of the curve passing through the point (1, -2) when the tangent at any point is given by $\frac{y(x+y^3)}{x(y^3-x)}$. 5
 - b) Find the distance of Geosynchronous orbits from Earth's surface in terms of earth's mass M, gravitational constant G and earth's radius R. 5
 - c) Calculate the torque necessary to produce a twist of one radian in wire of length 'L', modulus of rigidity η and radius 'r'. 5

- d) Write down the Lorentz Transformation equation. Using them, obtain the rule for length contraction. 5

GROUP-C

3. Answer any two questions: 10×2=20
- a) i) Considering the differential equation of an orbit, derive Newton's law of gravitation.
 - ii) Show that the angular momentum of a planet revolving round the sun remains constant. 5+5
 - b) i) The differential equation for a one dimensional damped harmonic oscillator is given by $m\frac{d^2x}{dt^2} + K\frac{dx}{dt} + Sx = 0$.
Explain the significance of each term in the equation.
 - ii) Solve the equation for critically damped condition. 2+8
 - c) i) If $\vec{A} = 3t^2\hat{i} = (t+4)\hat{j} + (t^2-24)\hat{k}$ and $\vec{B} = (\sin t\hat{i} + Se^{-t}\hat{j} - 3\cos t\hat{k})$
find $\frac{d^2}{dt^2}(\vec{A} \times \vec{B})$ at $t=0$.

- ii) Find out the equation of motion of a Rocket with variable mass when the Rocket is out of any external force.

5+5

- d) i) Prove that gravitational force is conservative.

- ii) Three particles of masses 4Kg, 3Kg, and 2Kg are at the points (2, 0, -1), (1, 1, 3) and (3, -1, 0) respectively. Find the co-ordinates of the centre of mass.

- iii) Suppose a spaceship heading directly towards the Earth at half the speed of light sends a signal to us on a laser-produced beam of light. Given that the light leaves the ship at speed c as observed from the ship, calculate the speed at which it approaches the Earth.

5+3+2

OPTION-B

PHYS(G)-CC-T-01(B)

(Mathematical Physics-I)

GROUP-A

1. Answer any five questions:

2×5=10

- a) Evaluate $\lim_{x \rightarrow 1} \frac{x^{10} - 1}{x^5 - 1}$

- b) State the order and degree of the differential equation

$$\frac{d^3 y}{dx^3} + \left(\frac{dy}{dx} \right)^2 + x^3 y = 0$$

- c) Check whether the three vectors $\hat{i}, \hat{i} + \hat{j}, \hat{i} + \hat{j} + \hat{k}$ are linearly independent.

- d) Check whether $dw = 2xy dx + x^2 dy$ is an exact differential.

- e) If $\vec{A}$ is a constant vector, find $\vec{\nabla}(\vec{A} \cdot \vec{r})$.

- f) Show that $\delta(ax) = \frac{1}{a} \delta(x)$, where $a > 0$.

- g) Prove that $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$

- h) Using Gauss' divergence theorem, show that

$$\iiint_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dV = \oiint_S (\phi \vec{\nabla} \psi - \psi \vec{\nabla} \phi) \cdot d\vec{S}$$

where $\phi(x, y, z)$ and $\psi(x, y, z)$ are two scalar functions and the surface integral is over the surface S enclosing the volume V .

GROUP-B

2. Answer any two questions: $5 \times 2 = 10$

a) Sketch the function $e^x, e^{-x}, e^{-|x|}$ for $-1 \leq x \leq 1$. Explain whether the function $e^{-|x|}$ is differentiable at $x = 0$.

b) Solve the equation

$$y'' + 6y' + 8y = 0$$

Subject to the condition $y = 1, y' = 0$ at $x = 0$

where $y' \equiv \frac{dy}{dx}$ and $y'' \equiv \frac{d^2y}{dx^2}$.

c) Considering two position vectors in three dimensions, show that their scalar product remains invariant under the rotation of co-ordinate system about z-axis.

d) Express the vector field $\vec{a} = yz\hat{i} - y\hat{j} + xz^2\hat{k}$ in cylindrical polar coordinates, and hence calculate its divergence. Show that the same result is obtained by evaluating the divergence in Cartesian coordinates. $3+2$

GROUP-C

Answer any two questions:

$10 \times 2 = 20$

3. a) Find the Taylor series expansion of $\sin x$ about $x = \pi$, giving the first two non-zero terms.

b) Suppose that the temperature T at any point (x, y, z) is given by

$$T(x, y, z) = x^2 - y^2 + yz + 373.$$

In which direction is the temperature increasing most rapidly at $(-1, 2, 3)$? What is the maximum rate of change of temperature at that point?

c) If S is any closed surface enclosing a volume V and $\vec{A} = ax\hat{i} + by\hat{j} + cz\hat{k}$.

$$\oint_S \vec{A} \cdot d\vec{S} = (a+b+c)V. \quad 3+(1+3)+3$$

4. a) Prove that $\vec{\nabla} \times (\phi \vec{V}) = (\vec{\nabla} \phi) \times \vec{V} + \phi (\vec{\nabla} \times \vec{V})$ for a scalar field $\phi(x, y, z)$ and a vector field $\vec{V}(x, y, z)$. Now take $\vec{V}$ to be a non-zero constant vector field $\vec{C}$ and use Stoke's theorem to prove that,

$$\oint_C \phi d\vec{r} = \iint_S d\vec{S} \times \vec{\nabla} \phi$$

where the closed curve C is the boundary of the surface S .

- b) Find the values of λ, μ, ν so that the vector $\vec{F} = (x + \lambda y + 4z)\hat{i} + (2x - 3y + \mu z)\hat{j} + (\nu x - y + 2z)\hat{k}$ is conservative. Find also the scalar function $\phi(x, y, z)$ such that $\vec{F} = \vec{\nabla}\phi$. (3+3)+4

5. a) Using Green's theorem evaluate $\int_C (x^2 y dx - x^2 dy)$ where C is boundary described counter-clock wise of the triangle with vertices (0, 0), (1, 0) (1, 1)

If $\vec{F} = (2x^2 - 3z^2)\hat{i} - 2xy\hat{j} - 4x\hat{k}$ find $\iiint_V \vec{\nabla} \cdot \vec{F} dV$ where V is the region bounded by the co-ordinate planes and the plane $2x + 2y + z = 4$.

- b) Perform the integral

i) Show that the infinitesimal volume element in Spherical Polar Coordinate system (r, θ, ϕ) is $r^2 \sin\theta dr d\theta d\phi$.

ii) Verify the divergence theorem for $\vec{A} = 4xz\hat{i} + y^2\hat{j} + yz\hat{k}$ and a cube bounded by the planes

$$x = 0, x = 1, y = 0, y = 1, z = 0 \text{ and } z = 1.$$

- c) Prove that $\oint u \vec{\nabla} v \cdot d\vec{r} = -\oint v \vec{\nabla} u \cdot d\vec{r}$. 6+2+2

OPTION-C

PHYS-G-CC-T-01(C)

(Electricity and Magnetism)

GROUP-A

1. Answer any five questions: 2×5=10
- Define polarization vector of a dielectric. What is its physical significance?
 - A point charge 'q' is kept at a corner of a cube. Determine the flux of the electric field due to 'q' through the three surfaces of the cube which do not meet at 'q'.
 - A point charge $Q = 30 \times 10^{-9}$ C is located at the origin in Cartesian coordinates. Find the electric flux density $\vec{D}$ at a point (1, 3, 4).
 - Define Electrical susceptibility and Dielectric constant.
 - Define Poynting vector.
 - Show that dipole moment of a charge distribution is independent of origin chosen if total charge is zero.
 - An electron is moving in a circular orbit of radius r with speed v. If we consider that it constitutes a steady current, find its magnitude.
 - If magnetic vector potential $\vec{A} = e^{-x} \sin y \hat{i} + (1 + \cos y) \hat{j}$, calculate the magnetic induction.

GROUP-B

2. Answer any two questions: 5×2=10

- a) A long hollow metal cylinder with inner radius 'a' and outer radius 'b' has a length 'l'. Show that the self-inductance of the cylinder is

$$L = \frac{l\mu_0}{2\pi} \ln \frac{b}{a}$$

Show that, a current placed in a magnetic field

$\vec{B}$ experiences a force $\vec{F} = I\vec{dl} \times \vec{B}$, where the symbols have their usual meanings.

Find the inductance in Henry of a straight coil of 100 turns, wound on 25cm long paper tube having 4cm radius. 2+2+1

- b) State and obtain the integral form of Gauss's law in a dielectric. Write down the boundary conditions at the interface of two dielectrics of permittivities ϵ_1 and ϵ_2 . Two parallel plate capacitors, each of capacitance $40\mu\text{F}$, are connected in series. The space between the plates of one capacitor is filled with a dielectric material of dielectric constant $K=4$. What is the equivalent capacitance of the system? 2+1+2

- c) A charged particle moves with uniform velocity $\vec{v} = 4\hat{i} \text{ m/s}$ in a region where $\vec{E} = 20\hat{j} \text{ V/m}$ and $\vec{B} = B_0\hat{k} \text{ Wb/m}^2$. Determine B_0 such that the velocity of the particle remains constant.

Calculate the magnetic field at a distance r

from the axis of a very long solenoid with radius R and having N turns per unit length, and carrying a steady current I.

Two long parallel wires each carrying 1 A current and placed 10 cm apart. What is the force per unit length between the two wires? 2+2+1

- d) If A is a constant vector and $\vec{r} = ix + jy + kz$ prove that $\nabla(\vec{r} \cdot \vec{A}) = \vec{A}$

If $\vec{F} = (x+2y)\hat{i} + (2x^2 + xy)\hat{j}$ evaluate the line integral $\int \vec{F} \cdot d\vec{r}$ along the curve c in the xy plane having equation $y=x^2$ from the point (0,0) to (1,1). 2+3

GROUP-C

3. Answer any two questions: 10×2=20

- a) Show that the electric and magnetic energy densities are equal in the propagation of electromagnetic waves in vacuum. Derive the

continuity equation $\nabla \cdot \vec{J} = \frac{\partial \rho}{\partial t}$ from Maxwell's

equations. Discuss the origin of the displacement current in Maxwell's equations. 4+4+2

- b) An electromagnetic wave is travelling in a linear, homogeneous and isotropic conducting medium where there are no charges and external currents. Derive the wave equations for the fields.

Apply Gauss theorem calculate the electric field due to a uniformly charged sphere of radius R at points inside and outside the sphere. Represent the above result graphically. 4+4+2

- c) Calculate the electrostatic potential in free space due to a dipole. For a uniformly charged disc of radius a , find the electric field at a distance h ($h \gg a$) from the centre along the axis of the disc.

Show that the vector $\vec{E} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ represents an electric field. Find the corresponding electrostatic potential V , given that $V = V_0$ at $x=y=z=0$. 4+3+3

- d) Starting with the expression $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, where the symbols have their usual meanings, show that, the line integral of the magnetic induction around a closed path is equal to μ_0 times the total current enclosed by the path.

Determine the force acting on an electric dipole placed within an inhomogeneous electric field. Two magnetic media are separated by a plane interface. Establish a relation of angles between the normal to the boundary and the $\mathbf{B}$ fields on either side. 4+3+3

field. Two magnetic media are separated by a plane interface. Establish a relation of angles between the normal to the boundary and the B fields on either side.

4+3+3