

U.G. 2nd Semester Examination - 2022

STATISTICS

[HONOURS]

Course Code : STAT-H-CC-T-3

(Mathematical Analysis)

Full Marks : 75(60+15)

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meaning.*

1. Answer any **ten** questions: $2 \times 10 = 20$
- Let $c (\in A^c)$ be a boundary point of the set $A (\subset R)$. Show that c is a cluster point of the set A .
 - Show that , the set of rational numbers is countable.
 - Let A be a subset of and $B := \{-x : x \in A\}$. Establish a relation between $\sup A$ and $\inf B$.
 - Let $K = \{s + t\sqrt{2} : t, s \in \}$. Show that K satisfies the followings:
 - If $x_1, x_2 \in K$, then $x_1 + x_2 \in K$ and $x_1 x_2 \in K$.

ii) If $x \neq 0$ and $x \in K$ then $1/x \in K$.

- Construct a sequence $\{x_n\}$ such that $x_n \in$ c
 $\forall n \geq 1$ and $x_n \rightarrow 2$ as $n \rightarrow \infty$.
- Show that if a series $\sum_{n \geq 1} a_n$ is convergent then
 $\frac{1}{n} \sum_{k=1}^n k a_k \rightarrow 0$ as $n \rightarrow \infty$.
- Let the sequence $\{a_n\}$ converge to a . Show that the sequence $\{|a_n|\}$ of absolute values converges to $|a|$.
- Find the limit of the sequence $\{a_n\}$ where

$$a_n = \left(1 + \frac{2}{n}\right)^n, n \in .$$
- Define absolute convergence and conditional convergence of a real series with example(s).
- If the sequence $\{n^2 a_n\}$ is convergent then show that $\sum_{n \geq 1} a_n$ is convergent absolutely.
- Check the existence of the following limit:

$$\lim_{x \rightarrow 2} \sqrt{x+2}.$$
- Give an example of a real valued function which is continuous at exactly one point of its domain but discontinuous elsewhere.

[Turn Over]

- m) Define uniform continuity of a function.
 n) Stating necessary result(s), find the limit

$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x}.$$

- o) Evaluate the double integral:

$$\iint_D \frac{y^2}{x^2} dA$$

where D is a region in the first quadrant bounded by the curves

$$y = x, y = \frac{1}{x} \text{ and } y = \frac{3x}{1+x^2}.$$

- d) Define a locally bounded function. If $\lim_{x \rightarrow c} f(x) = \ell$ ($\ell \in \mathbb{R}$) then show that f is locally bounded at $x = c$. 1+4

- e) Show that the function $f(x) = \sqrt{x}$ is uniformly continuous on $(0, \infty)$. 5

- f) Using the technique of double integration, show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ where

$$\Gamma(p) = \int_0^\infty e^{-x} x^{p-1} dx. \quad 5$$

3. Answer any **two** questions: 10×2=20

- a) i) Define Cauchy sequence of real numbers. State its relation with convergent sequence.

Show that the sequence $a_n = \frac{\sin n}{\sqrt{n}}$, $n \geq 1$ is a Cauchy sequence.

- ii) Let $\{a_n\}$ be a sequence of positive real numbers such that for $n > 1$, $a_{n+1} < a_n + \frac{1}{(n+1)^2}$.

Show that the sequence is convergent.

(1+2+2)+5

2. Answer any **four** questions: 5×4=20

- a) State and prove the Archimedean property of real numbers. 5

- b) Cite examples of two functions such that one is (i) bounded but not continuous at each point of the domain, and another is (ii) continuous in the domain but not bounded. Justify your answers. 2+3

- c) State Lagrange's Mean Value theorem. Show that, for any differentiable function f , passing through the origin such that $1 \leq f'(x) \leq 2$ for all $x \in \mathbb{R}$, $x \leq f(x) \leq 2x$ when $x > 0$. 1+4

b) i) Show that

$$\int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx$$

is convergent if $m > 0$ and $n > 0$.

ii) Write down the importance of bijective transformation in view of multiple integral. Explain why orthogonal transformation is a bijective transformation. 5+(1+4)

c) i) Write Taylor's theorem with Lagrange's form of remainder.

ii) Maximize $f(x, y, z) = xyz$ subject to

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \text{ where } a, b, c > 0. \quad 4+6$$

[Internal Assessment: 15]
