

U.G. 1st Semester Examination - 2025

MATHEMATICS

[MINOR]

Course Code.: MATH-MI-T-1

(Algebra and Analytical Geometry)

[NEP-2020]

Full Marks : 40

Time : 2 Hours

*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**The notations and symbols have their usual meanings.*1. Answer any five questions: $2 \times 5 = 10$

- a) Give the polar representation of the complex number: $z = -1 - i$.
- b) Show that the equation $x^2 + 3xy + 2y^2 = 0$ represents a pair of straight lines.
- c) Show that $G = \{2^n : n = 0, \pm 1, \pm 2, \pm 3, \dots\}$ is a multiplicative group.
- d) Find the transformed form of the equation $\frac{x}{a} + \frac{y}{b} = 2$ when the origin is transferred to (a, b) .
- e) Find the equation of the sphere with centre at $(0, 0, 1)$ and radius 1.

[Turn over]

f) Define a relation ρ on Z by $a\rho b$ if and only if $3a+4b=7n$, for some integer n . Check whether ρ is transitive.

g) Find the condition so that the rank of $\begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix}$ is 3.

h) Find the value of $\alpha\beta+\alpha\gamma+\beta\gamma$ if α, β, γ are the roots of $x^3-6x^2+11x-6=0$.

2. Answer any two questions: $5 \times 2 = 10$

a) Find for what values of a, b the following system $x+4y+2z=1$,
 $2x+7y+5z=2b$,
 $4x+ay+10z=2b-1$, has a unique solution.

5

b) Expand $\cos^7 \theta$ in a series of cosines of multiples of θ .

5

c) Show that the equation to the pair of straight lines through the origin perpendicular to the pair of straight lines $ax^2+2hxy+by^2=0$, is $bx^2-2hxy+ay^2=0$.

5

d) If by a rotation of rectangular axes about the origin, the expression $(ax^2+2hxy+by^2)$ changes to $(AX^2+2HXY+BY^2)$, then prove that $a+b=A+B$ and $ab-h^2=AB-H^2$.

5

3. Answer any two questions: $10 \times 2 = 20$

a) i) If $\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma = 0$, then prove that $\cos(\beta+\gamma) + \cos(\gamma+\alpha) + \cos(\alpha+\beta) = 0$.

5

ii) Apply Descartes' rule of signs to examine the nature of the roots of the equation $x^4+2x^2+3x-1=0$.

5

b) i) Show that the map $f(x) = \frac{1}{1+e^{-x}}$ is a bijection from R to $(0,1)$.

4

ii) What do you mean by a subgroup of a group G ? Do you find a group structure in Z ? If so then check whether $H = \{x+2y: x, y \in Z\}$ is a subgroup of Z or not. If A, B are to subgroups of Z , then show that $A \cap B$ is also a subgroup of Z . Here $Z =$ set of all integers.

6

c) i) Show that the straight line $r \cos(\theta - \alpha) = p$ touches the conic $\frac{l}{r} = 1 + e \cos \theta$, if $(l \cos \alpha - ep^2)^2 + l^2 \sin^2 \alpha = p^2$.

6

- ii) Find the values of b and g such that the equation $4x^2 + 8xy + by^2 + 2gx + 4y + 1 = 0$ represents a conic having infinitely many centres. 4
- d) i) Find the equation of the sphere touching the three coordinate planes. 5
- ii) Find the equation of the right circular cone which contains the three positive coordinate axes. 5
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