

U.G. 1st Semester Examination - 2024

PHYSICS

[MAJOR]

Course Code : PHY-MJ-T-01
(Mathematical Physics-I)

[NEP-2020]

Full Marks : 40
Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any five questions : $2 \times 5 = 10$ a) Draw the curve of the function $y = x^2 - 3x - 4$.b) Find $\lim_{x \rightarrow 0} \frac{\tan x}{x}$.c) Find the particular integral of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$.d) Find the moment of a force $F = 3i + 2j + 2k$ acts at a point $2i + 3j + 4k$ about the origin.e) Prove that $\delta(ax) = \frac{1}{a} \delta(-x)$.

f) Prove that eigen value of a Hermitian matrix is real.

[Turn over]

- g) Show that $r^n \bar{r}$ is irrotational.
- h) Prove that $\int_C \bar{r} \cdot d\bar{r} = 0$.
- i) If 3 dice is tossed at a time find the probability of getting 12 altogether.

GROUP-B

2. Answer any two questions: $5 \times 2 = 10$
- a) i) If $u = \sin^{-1} \frac{x^2 + y^2}{x + y}$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$.
- ii) Find the directional derivative of the function $xy^2 + yz^2$ at the point $(1, -1, 1)$ in the direction of $(3, 1, -1)$.
- b) Evaluate $\iint_S \bar{F} \cdot \hat{n} dS$ where $\bar{F} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$ where S is the surface of the cylinder $x^2 + y^2 = 16$ included in the 1st octant between $z = 0$ and $z = 5$.
- c) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$.
- d) Solve the differential equation $(D^2 - 2D + 1)y = xe^x \sin x$.
- e) Find whether $(4, -2, 1)$; $(5, 1, 6)$; $(2, 2, -5)$ and $(3, 5, 0)$ lie in a plane?

GROUP-C

Answer any two questions : $10 \times 2 = 20$

3. a) Is the function $|x|$ continuous about $x = 1$?
- b) Find the derivative of $\sec^{-1}(\frac{1}{2x^2-1})$ with respect to $\sqrt{1-x^2}$ at $x = \frac{1}{2}$.
- c) Find ∇r where $\bar{r}$ is the position vector. $2+5+3$
4. a) Find Curl $\bar{E}$ where $\bar{E} = \frac{1}{4\pi\epsilon r^2} Q \hat{r}$ an electric field vector. Q is the point charge and $\bar{r}$ is the position vector.
- b) If a force $\bar{F} = (2x + 3)\hat{i} + xy\hat{j} + (zx - y)\hat{k}$, find the value of $\oint_C \bar{F} \cdot d\bar{r}$ along C where C is the straight line from $(0, 0, 0)$ to $(1, 0, 0)$; then to $(1, 0, 0)$ and then to $(1, 2, 1)$. $5+5$
5. a) Solve the differential equation $[(x+1)e^x - e^y]dx - xe^y dy = 0$ if $y(1) = 0$.
- b) Find the eigenvalue and eigenvectors of the matrix $\begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{vmatrix}$. $5+5$
6. a) Using Green's theorem evaluate $\int_C (x^2 y dx - x^2 dy)$ where C is boundary describe counterclockwise path along the triangle with vertices $(0, 0)$; $(1, 0)$; $(1, 1)$.

- b) The direction of two unit vector $\mathbf{n}_1$ and $\mathbf{n}_2$ given by spherical polar coordinates (α, β) and (γ, δ) respectively where α and β are the polar angle and azimuthal angle. Show that the angle φ between unit vector $\mathbf{n}_1$ and $\mathbf{n}_2$ is given by $\cos\varphi = \cos\alpha \cos\gamma + \sin\alpha \sin\gamma \cos(\beta - \delta)$.

5+5
