

**U.G. 5th Semester Examination - 2021****STATISTICS****[HONOURS]****Course Code : STAH-CC-T-11****(Statistical Inference-II)**

Full Marks : 50(40+10)

Time :  $2\frac{1}{2}$  Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meanings.*

1. Answer any **five** questions:  $2 \times 5 = 10$
- Define simple hypothesis and composite hypothesis with one example for each.
  - What is Type II error in connection with hypothesis testing? How is it related to power?
  - Define ASN in connection with sequential testing.
  - Define an UMP test for a simple null hypothesis against a composite alternative.
  - What is size of a test?
  - What is the advantage of using an SPRT over a fixed-sample test?
  - What is meant by a stopping rule in connection with sequential testing?

h) What is randomized test?

2. Answer any **two** questions:  $5 \times 2 = 10$
- For testing a simple null hypothesis against a simple alternative, show that the Neyman-Pearson test is equivalent to the likelihood ratio test.
  - Derive the MP size a randomized test for testing  $H_0 : \theta = \theta_0$  against  $H_1 : \theta = \theta_1 (> \theta_0)$  on the basis of a random sample of size  $n$  drawn from a Poisson distribution with parameter  $\theta$ .
  - Discuss the properties of likelihood ratio tests.
  - For the SPRT with stopping bounds  $A$  and  $B$ ,  $A > B$ , and strength  $(\alpha, \beta)$  show that  $A \leq \alpha^{-1}(1 - \beta)$  and  $B \geq (1 - \alpha)^{-1}\beta$ .
3. Answer any **two** questions:  $10 \times 2 = 20$
- State Neyman-Pearson Fundamental Lemma and prove its sufficiency part. Is the test obtained by this Lemma unique?
  - Derive the size  $\alpha$  likelihood ratio test for testing  $H_0 : \theta = \theta_0$  against  $H_1 : \theta > \theta_0$  on the basis of a random sample of size  $n$  drawn from a normal population with unknown mean  $\theta$  and known standard deviation  $\sigma$ . What optimum property is enjoyed by this test?

*[Turn over]*

- c) i) Suppose  $X$  is an observation from the distribution with p.d.f.

$$f(x|\theta) = \frac{1}{\pi} \frac{1}{1+(x-\theta)^2}; -\infty < x < \infty, -\infty < \theta < \infty.$$

Show the test  $\phi(x) = 1$  if  $1 < x < 3$ ; and 0 otherwise, is a best test of its size for testing  $H_0: \theta = 0$  against  $H_1: \theta = 1$ . 7

- ii) Define power function. What is power curve? 3

- d) Derive the SPRT of strength  $(\alpha, \beta)$  for testing  $H_0: \theta = \theta_0$  against  $H_1: \theta = \theta_1$  for a normal distribution with mean  $\theta$  and standard deviation 2. Write down the OC function of this test.

*[Internal Assessment: 10]*

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