

U.G. 6th Semester Examination - 2025

MATHEMATICS

[PROGRAMME]

Discipline Specific Elective (DSE)

Course Code : MATH-G-DSE-T-02(A)&(B)

[New Syllabus]

Full Marks : 60

Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer all the questions from Selected Option.

OPTION-A

MATH-G-DSE-T-02A

(Dynamics of a Particle)

1. Answer any ten questions: $2 \times 10 = 20$
- a) A man starts on a long walk at 4km per hour and his speed at any instant is inversely proportional to $(x + 10)$. Find the time he takes to walk 20 km.
- b) The velocity of a particle at a distance x from a fixed point O is given by $v^2 = a - bx^2$, where a and b are constants. Show that the motion is simple harmonic motion and determine its period.

[Turn Over]

- c) A motor car driven with a constant force at all speeds meets with air resistance proportional to the square of the velocity. If u be the maximum speed of the car, then show that its acceleration when running at a speed v varies as $u^2 - v^2$.
- d) What do you mean by conservative force? Give two examples of forces among which one is conservative but other one not.
- e) What do you mean by impulsive force? State the principle of conservation of linear momentum.
- f) A body of mass 10 pounds moving with a velocity of 12 feet per second meet a body of mass 8 pounds moving in the opposite direction with a velocity of 4 feet per second. If they collide into one body, then find the velocity of the compound body.
- g) A ball is dropped vertically on a fixed horizontal plane from a height of 100 ft. If the coefficient of restitution be 0.4, find the velocities after the first and second rebound.
- h) Prove that the acceleration of a particle moving in a curve with uniform speed is $\frac{v^2}{\rho}$.

- i) The co-ordinates of a moving point at time t are given by $x = a(2t + \sin 2t)$, $y = a(1 - \cos 2t)$. Prove that its acceleration is constant.
- j) A particle describes an equiangular spiral $r = ae^\theta$ in such a way that its acceleration has no radial component. Prove that its angular velocity is constant.
- k) Show that in a central orbit, angular momentum about the centre of force remains constant.
- l) Prove that at an apse, the tangent to the orbit is perpendicular to the radius vector.
- m) Prove that a body projected from the earth's surface with a speed exceeding 7 miles per second will not, in general, return to the earth (assume that the radius of the earth is 4000 miles).
- n) State the Newtonian law of gravitation.
- o) A particle describes a curve whose equation is $r = a \sec^2 \frac{1}{2} \theta$ under a force to the pole. Show that the force varies inversely as the square of the distance.

2. Answer any four questions:

5×4=20

- a) A particle of mass m moves in a straight line OX under a force $m\mu \times (\text{distance})$ directed towards a point A which moves in the straight line OX with a constant acceleration α . Show that the motion of the particle is simple harmonic of period $\frac{2\pi}{\sqrt{\mu}}$ about a moving centre which is always at a distance $\frac{\alpha}{\mu}$ behind A.
- b) A particle of mass m is projected vertically upwards under gravity, the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is $\frac{v^2}{g}[\lambda - \log(1 + \lambda)]$, where V is the terminal velocity of the particle and λV is the initial velocity.
- c) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to height h . show that the velocity

of recoil of the gun is $\left\{ \frac{2m^2 gh}{M(m + M)} \right\}^{1/2}$.

- d) A particle falls to the ground from a height h . If e be the coefficient of restitution, then show that the whole distance described by the particle before it has finished rebounding is $h \frac{1+e^2}{1-e^2}$.
- e) A particle is acted on by a force parallel the axis of y whose acceleration (always towards the axis of x) is μy^{-2} , and when $y = a$, it is projected parallel to the axis of x with velocity $\sqrt{\frac{2\mu}{a}}$; prove that it will describe a cycloid.
- f) A particle describes the equilibrium spiral $r = ae^{\theta}$ in such a manner that the radial acceleration is zero. Prove that the speed and the magnitude of acceleration are each proportional to r .

3. Answer any two questions:

10×2=20

- a) A particle of mass m moves under a central attractive force $m\mu(5u^3 + 8c^2u^5)$ and is projected from an apse at a distance c with velocity $3\sqrt{\mu/c}$; prove that the orbit is $r = a \cos \frac{2}{3}\theta$, and that it will arrive at the origin after a time $\frac{\pi c^2}{8\sqrt{u}}$.

7+3

- b) i) If v_1 and v_2 are the linear velocities of a planet when it is respectively nearest and farthest from the sun, prove that $(1-e)v_1 = (1+e)v_2$, where e is the eccentricity of the elliptic path of the planet. 5

- ii) A particle moving in a straight line is acted on by a force which works at a constant rate and changes its velocity from u to v in passing over a distance x . Prove that the time taken is $\frac{3(u+v)x}{2(u^2+uv+v^2)}$. 5

- c) i) A particle leaves the origin with velocity u . If it moves with an acceleration $\frac{\mu}{v^2}$, v being the velocity at any instant, show that the distance x described in time t is given by $4\mu x = 3(\mu t + u^2)^{\frac{4}{3}} - u^4$. 5
- ii) A point moves on a plane with constant linear velocity ωa its angular velocity about the pole is $\frac{\omega r}{a}$; show that its acceleration is equal to $3\omega^2 r$. 5

OPTION-B
MATH-G-DSE-T-02B
(Numerical Methods)

1. Answer any ten questions: 2×10=20
- Find the number of significant figures in $X_A = 11.2843$ given its relative error as 01×10^{-3} .
 - Find the absolute error and relative error in taking $\pi = 3.141593$ as $\frac{22}{7}$.
 - Show that $\Delta = \Delta E$.
 - State fundamental theorem of finite difference calculus.
 - Show that any divided difference of a constant is zero.
 - State Stirling's and Bessel's interpolation formula (without error).
 - State the advantage of Lagrange's interpolation.
 - Explain the principle of numerical integration.
 - Why the Newton-Raphson method called the method of tangent?
 - Evaluate $\Delta(\log x)$.
 - What do you mean by the order of convergence for solving algebraic equation?

l) Write down the error formula for composite trapezoidal rule.

m) What do you mean by the degree of precision of a quadrature formula?

n) Is it possible to find numerically least eigen value for a matrix A by power method? Discuss.

o) What do you mean by the diagonally dominant for system of linear equations?

p) State the basic principle of Iteration method.

2. Answer any four questions: $5 \times 4 = 20$

a) Establish Newton's forward interpolation formula. When is this formula used?

b) Establish a general quadrature formula for equidistant ordinates and deduce Simpson's $1/3^{\text{rd}}$ rule for numerical integration.

c) State the Newton-Raphson formula for solving a transcendental equation and explain the limitation of using Newton-Raphson formula.

d) Show that the total number of multiplications and divisions required to solve a system of n linear equations in n unknowns by Gauss

elimination method is $\frac{1}{3}n^3 + n^2 - \frac{1}{3}n$.

e) Find the Lagrange interpolation polynomial to fit the following data:

x	3	7	9	10
$y = f(x)$	168	120	172	63

f) Solve $\frac{dy}{dx} = x^3 + xy^2$, $y(0) = 1$ by Euler's modified method and the values of $y(0.2)$ and $y(0.4)$ by taking $h = 0.2$.

3. Answer any two questions: $10 \times 2 = 20$

a) Describe Gauss elimination method for numerical solution of a system of linear algebraic equations. Examine the necessity of the pivoting process involved in it. $7+3$

b) Describe Newton's divided difference formula for interpolation formula with remainder. Hence deduce Newton's backward difference interpolation formula from this method. $7+3$

c) Deduce the method of iteration for numerical solution of an equation of the form $x = \phi(x)$ and obtain the condition of convergence. $6+4$