

U.G. 4th Semester Examination - 2024**STATISTICS****[HONOURS]****Generic Elective Course (GE)****Course Code : STAT-H-GE-T-4****(Introductory Probability)**

Full Marks : 50(40+10)

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meaning.*

1. Answer any five questions: $2 \times 5 = 10$
- a) Give the axiomatic definition of probability.
 - b) Suppose a random variable X has a Poisson distribution with $P(X=0) = e^{-5/2}$. What is the mode of X ? When does binomial distribution tend to Poisson distribution?
 - c) Write down the p.m.f. of a discrete random variable having the 'loss of memory property'. What is the mean of this distribution?

[Turn Over]

- d) An urn contains two blue and three red marbles. Three marbles are drawn from this urn at random one by one without replacement and their colours are noted. Write down the sample space for this experiment.
- e) Suppose that A, B and C are mutually independent events and that $P(A) = 0.5$, $P(B) = 0.8$ and $P(C) = 0.9$. Find the probability that exactly two of the three events occur.
- f) Suppose X is a random variable with density function $f(x) = \frac{1}{B(a,b)} x^{a-1} (1-x)^{b-1}$ ($a, b > 0$), for $0 < x < 1$; and 0 otherwise. Find $E(X^2)$.
- g) State Lindeberg-Levy Central Limit Theorem.
- h) Find the expectation of Gamma distribution.
2. Answer any two questions: $5 \times 2 = 10$
- a) A man draws 3 balls from a bag containing 4 black and 7 green balls. If he is to receive Rs. 20 for every black ball which he draws and Rs. 10 for every green ball. What is his expectation?

- b) The p.d.f. of a random variable X is

$$f(x) = \begin{cases} 0 & \text{for } x \leq -a \\ \frac{a+x}{a^2} & \text{for } -a < x \leq 0 \\ \frac{a-x}{a^2} & \text{for } 0 < x \leq a \\ 0 & \text{for } x > a. \end{cases}$$

Find the cumulative distribution function of X.

- c) Find the recursion relation involving the central moments of binomial distribution.
- d) Find mean deviation about mean of normal distribution.

3. Answer any two questions: $10 \times 2 = 20$

- a) i) State and prove the weak law of large numbers (WLLN).
- ii) Let $P[X_k = \pm 2^k] = 2^{-(2k+1)}$, $P[X=0] = 1 - 2^{-2k}$, $k=1, 2, \dots$

Examine whether WLLN holds for the sequence $\{X_k\}$. $5+5$

- b) i) Find the mode of Poisson distribution.
- ii) Show that for a Poisson distribution with unit mean, mean deviation about mean is $2/e$ times the standard deviation. $5+5$

c) i) Let X be a random variable with p.m.f.
 $f(x) = p(1 - p)^x$, $x = 0, 1, \dots$; $0 < p < 1$.
 Find $E(X)$.

ii) Show that for any two events A and B
 defined on a sample space

$$P(A) + P(B) - 1 \leq P(A \cap B) \leq \min\{P(A), P(B)\}.$$

5+5

d) i) Let X follow normal distribution with
 mean μ and variance σ^2 . Show that $\mu_{2n} =$
 $1.3.5 \dots (2n-1)\sigma^2$ where μ_r is the r^{th} order
 central moment of X .

ii) In a class of 75 students, 15 were
 considered to be very intelligent, 45 as
 medium and the rest below average. The
 probability that a very intelligent student
 fails in a viva-voce examination is 0.005,
 the medium student failing has a
 probability 0.05 and the corresponding
 probability for a below average student is
 0.15. If a student is known to have passed
 the viva-voce examination, what is the
 probability that he is below average?

5+5

[Internal Assessment: 10]