

U.G. 3rd Semester Examination-2024

STATISTICS

[MAJOR]

Course Code : ST-MJ-T-3

(Linear Algebra and Numerical Analysis)

[NEP-2020]

Full Marks : 40

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meaning.*

1. Answer any five questions: $2 \times 5 = 10$
- a) Define diagonal matrix and triangular matrix.
 - b) Show that matrix addition is commutative.
 - c) Define dimension of a vector space.
 - d) If $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{pmatrix}$, what are the eigenvalues of A ?
 - e) What is meant by absolute error and relative error?
 - f) Round-off the following numbers up to 4-decimal places: 56.243827, 0.235082, 0.560012, 40.35856.

[Turn over]

- g) Establish the relation $\Delta = E - 1$.
- h) Explain numerical differentiation.
2. Answer any two questions: $5 \times 2 = 10$
- a) If V be a real vector space with $\{\alpha, \beta, \gamma\}$ as a basis, prove that the set $\{\alpha + \beta + \gamma, \beta + \gamma, \gamma\}$ is also a basis for V .
- b) If A be a square matrix, show that $A + A^T$ is symmetric and $A - A^T$ is skew symmetric.
- c) Prove that $\nabla E = E \nabla = -\Delta$.
- d) Explain trapezoidal rule in the context of numerical integration.

3. Answer any two questions: $10 \times 2 = 20$
- a) i) Prove without expanding
- $$\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix} = 0$$
- ii) If A be an orthogonal matrix prove that A is non-singular and $\det A = \pm 1$. $5+5$
- b) i) Suppose S be the subset of $\mathbb{R}^3$ defined by $S = \{(x, y, z) \in \mathbb{R}^3 : y = z = 0\}$. Show that S is a subspace of $\mathbb{R}^3$.
- ii) Prove that the eigenvectors of a square matrix A over a field F corresponding to its two distinct eigenvalues are linearly independent. $5+5$

- c) Derive Newton's forward interpolation formula along with its error term.
- d) i) Describe the method to iteration to find a real root of a transcendental equation $f(x) = 0$ and derive the condition for the convergence of this method.
- ii) Write a note on Stirling's approximation to $n!$. $6+4$