

U.G. 3rd Semester Examination - 2025**MATHEMATICS****[MINOR]****Course Code : MATH-MI-T-02****(Calculus & Differential Equations)****[NEP-2020]**

Full Marks : 40

Time : 2 Hours

*The figures in the right-hand margin indicate marks.**The notations and symbols have their usual meanings.*

1. Answer any five questions: $2 \times 5 = 10$
- a) State Leibnitz's theorem of successive derivative.
 - b) Show that $f(x) = |x|$ is continuous at $x = 0$ but not differentiable at $x = 0$.
 - c) Find the n th derivative of $y = \log(ax+b)$
 - d) State Maclaurin's theorems with Lagrange's forms of remainder.
 - e) Write Taylor's infinite series of function $\log(1+x)$ with restriction.
 - f) What do you mean by General and Singular solution of a differential equation?

[Turn over]

g) Evaluate the following limit:

$$\lim_{n \rightarrow \infty} (ne^n).$$

h) Discuss the existence of extreme value of

$$f(x) = x^{\frac{1}{3}}.$$

2. Answer any two questions:

$$5 \times 2 = 10$$

a) If $y = \sin(m \sin^{-1} x)$, then show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0$.

b) Find the maximum value of $x^m y^n$ where x and y are positive and $x + y = k$.

c) Find the singular solution of the equation $p^2 + 4xp - 4y = 0$. Where $p = dy/dx$.

d) Solve $(D^2 + a^2)y = \sin(ax) + xe^{2x}$.

3. Answer any two questions:

$$10 \times 2 = 20$$

i) a) Solve using Variation of Parameters

$$\frac{d^2 y}{dx^2} - y = 2x^2 - x - 3.$$

b) Solve $\frac{d^2 y}{dt^2} + x = \cos t$, $\frac{d^2 x}{dt^2} + y = \sin t$.

ii) a) Prove that $\int_0^{\frac{\pi}{2}} \sin^{\frac{3}{2}} x \cos^3 x dx = \frac{8}{45}$.

b) Show that the radius of curvature at the point (r, θ) of the curve $r^2 \cos 2\theta = a^2$ is $\frac{r^3}{a^2}$.

iii) a) Solve:

$$(5 + 2x)^2 \frac{d^2 y}{dx^2} - 6(5 + 2x) \frac{dy}{dx} + 8y = 6x.$$

b) Evaluate $\lim_{x \rightarrow a} \frac{\log(x-a)}{\log(e^x - e^a)}$.

iv) a) If ρ_1, ρ_2 be the radii of curvature at the extremities of any chord of the cardioid $r = a(1 + \cos \theta)$, which passes through the pole, then prove that $\rho_1^2 + \rho_2^2 = \frac{16}{9} a^2$.

b) If $V = \log_e(x^3 + y^3 + z^3 - 3xyz)$, then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) V = \frac{-3}{(x+y+z)^2}$.