

U.G. 6th Semester Examination - 2024

MATHEMATICS

[PROGRAMME]

[New Syllabus]

Discipline Specific Elective (DSE)

Course Code : MATH-G-DSE-T-02(A)&(B)

Full Marks : 60

Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer all the questions from Selected Option.

OPTION-A

MATH-G-DSE-T-02A

(Dynamics of a particle)

1. Answer any ten questions: $2 \times 10 = 20$

- i) A particle moves in a straight line according to the law $v^2 = 4(x \sin x + \cos x)$, where v is the velocity at a distance x from a fixed point on its path. Find the acceleration.

[Turn Over]

- ii) A particle moves in a straight line so that after t seconds its distance from a fixed point O on the line is given by $s = (t-1)^2(t-2)$. Find the distance from O when its velocity is zero.
- iii) The velocity of a particle distance x from a fixed point O is given by $v^2 = a - bx^2$, where a and b are constants. Show that the motion is simple harmonic and determine its period and amplitude.
- iv) A motor car driven with a constant force at all speeds meets with air resistance proportional to the square of the velocity. If u be the maximum speed of the car, then show that its acceleration when running at a speed v varies as $u^2 - v^2$.
- v) A particle moves with a simple harmonic motion, its position of rest being at a distance a from the centre. Find, by the principle of energy, the velocity at the centre.
- vi) Write the definition of Potential Energy (P.E.) and state the Principle of Energy.
- vii) Find the work done by the gravity on a stone having a mass of 70 grams, during the tenth second of its fall from rest.

- viii) What do you mean by impulsive force and state the Principle of Conservation of Linear Momentum.
- ix) The position of a moving particle at time t is given by $x = a \cos nt, y = a \sin t$. Find the path of the particle, its velocity and acceleration.
- x) A particle moving in a circle of radius a , what will be the expression of acceleration components along the radial and tangential directions in polar coordinates.
- xi) What do you mean by apse on a central orbit? Prove that at an apse $p=r$ (symbols are used in usual notations).
- xii) State three Kepler's laws of Planetary Motion.
- xiii) A particle falls from a height h in time t upon a fixed horizontal plane. it rebounds and reaches the maximum height h' in time t' . Show that $t' = et$ and $h' = e^2 h$, e being the coefficient of elasticity.
- xiv) If the angular velocity of a moving particle about a fixed origin be constant, then show that the transverse acceleration varies as its radial.

- xv) A particle describe parabola with uniform speed. Show that its angular velocity about the focus S, at any point P, varies inversely as $(SP)^{\frac{3}{2}}$.

2. Answer any four questions: $5 \times 4 = 20$

- i) A particle of mass m is acted on by a force $m\mu\left(x + \frac{a^4}{x^3}\right)$ towards the origin. If it starts from

rest at a distance a , then show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$.

- ii) A particle describes the equiangular spiral $r = ae^{\theta}$ in such a manner that the radial acceleration is zero. Prove that the speed and the magnitude of acceleration are each proportional to r .

- iii) A spherical drop of liquid falling freely in a vapour acquires mass by condensation at a constant rate k . Show that the velocity after falling from rest in time is $\frac{1}{2}gt\left(1 + \frac{M}{M+kt}\right)$, where M is the initial mass of the drop.

- iv) If an engine of Horse Power H draws a train of W tons up a plane of inclination θ with a uniform velocity of v ft/sec, against a resistance of R lbs-weight per ton, prove that

$$H = \frac{Wv}{550}(R + 2240\sin\theta).$$

If the engine draws the train down the plane, prove that

$$H = \frac{Wv}{550}(R - 2240\sin\theta).$$

- v) A particle falls to the ground from a height h . If e be the coefficient of restitution, then show that the whole distance described by the particle before it has finished rebounding is $h \frac{1+e^2}{1-e^2}$ and

$$\text{that the whole time taken is } \sqrt{\frac{2h}{g}} \frac{1+e}{1-e}.$$

- vi) A particle moves with a central acceleration $\mu/(\text{distance})^2$, it is projected with velocity V at a distance R . Show that its path is a rectangular hyperbola if the angle of projection is $\sin^{-1} \mu / \{VR(V^2 - 2\mu/R)^{1/2}\}$.

3. Answer any two questions: $10 \times 2 = 20$

- i) a) If a be the amplitude and T be the period of particle executing S.H.M., in a straight line, show that the time taken by the

particle to travel a distance x from the centre of force is $\frac{T}{2\pi} \sin^{-1}(x/a)$ and the velocity in that position is $\frac{T}{2\pi} \sqrt{a^2 - x^2}$.

- b) A particle is projected with velocity V along a smooth horizontal plane in a medium whose resistance per unit mass is μ times the cube of the velocity. Show that the distance it describes in time t is $\frac{1}{\mu V} [\sqrt{1+2\mu V^2 t} - 1]$ and that its velocity then is $V / \sqrt{1+2\mu V^2 t}$.

- ii) a) A uniform chain of length $2a$ is hung over a smooth peg so that the length of it on the two sides are $(a+b)$ and $(a-b)$. Motion is allowed to ensue. Show that the chain leaves the peg at the end of time

$$\sqrt{\frac{a}{g}} \log \left(\frac{a + \sqrt{a^2 - b^2}}{b} \right).$$

- b) A shell of mass m is ejected from a gun of mass M by an explosion which generates kinetic energy E . Prove that the initial

velocity of the shell is $\sqrt{\frac{2mE}{(M+m)m}}$.

- iii) a) A particle moves freely in a parabolic path given by $y^2 = 4ax$, under a force always perpendicular to its axis. Find the law of force.

- b) A particle, moving with a central acceleration $\mu / (\text{distance})^2$, is projected from an apse at a distance a with a velocity V . If $V^2 < \mu / a^2$, find the path.

OPTION-B

MATH-G-DSE-T-02B

(Numerical Methods)

1. Answer any ten questions: $2 \times 10 = 20$
- a) Find the number of significant figures in $X_A = 1.8924$ given its relative error as 01×10^{-2} .
 - b) State fundamental theorem of finite difference calculus.
 - c) Find the absolute error and relative error in taking $\pi = 3.141593$ as $\frac{22}{7}$.
 - d) Show that $\Delta \log f(x) = \log \left\{ 1 + \frac{\Delta f(x)}{f(x)} \right\}$.
 - e) State Stirlings and Bessel's interpolation formula (without error).
 - f) Show that any divided difference of a constant is zero.
 - g) State the advantage of Lagrange's interpolation.
 - h) Evaluate $\Delta(\tan^{-1} x)$.
 - i) Explain the principle of numerical differentiation.

- j) Why the Newton-Raphson method called the method of tangent?
- k) Write down the error formula for composite Simpson's 1/3rd rule.
- l) What do you mean by the degree of precision of a quadrature formula?
- m) What do you mean by the order of convergence for solving algebraic equation?
- n) Is it possible to find numerically least eigen value for a matrix A by power method? Discuss.
- o) What do you mean by the diagonally dominant for system of linear equations?
- p) State the basic principle of Newton-Raphson method.

2. Answer any four questions: $5 \times 4 = 20$
- a) State the Newton-Raphson formula for solving a transcendental equation and explain the limitation of using Newton-Raphson formula.
 - b) Establish Newton's backward interpolation formula, When is this formula used?
 - c) Find the Lagrange interpolation polynomial to fit the following data:

i	0	1	2	3
x_i	0	1	2	3
$y = f(x_i)$	0	1.72	6.39	19.08

- d) Establish a general quadrature formula for equidistant ordinates and deduce Trapezoidal rule for numerical integration.
- e) To solve the non-linear equation $e^{-x} - \cos x = 0$ by fixed-point iteration method, the following fixed-point formulation may be considered:
- $x = -\ln(\cos x)$
 - $x = \cos^{-1}(e^{-x})$
- f) Show that the total number of multiplications and divisions required to solve a system of n linear equations in n unknowns by Gauss elimination method is $\frac{1}{3}n^3 + n^2 - \frac{1}{3}n$.
- g) Solve $\frac{dy}{dx} = x^2 + y^2$, $y(0)=1$ by Euler's modified method and the values of $y(0.2)$ and $y(0.4)$ by taking $h=0.2$.
3. Answer any two questions: $10 \times 2 = 20$
- a) Describe Newton's divided difference formula for interpolation formula with remainder. Hence deduce Newton's forward difference interpolation formula from this method.

$$7+3=10$$

- b) Establish Lagrange's polynomial interpolation formula. If $x_1, x_2, \dots, x_n$ be the interpolating points and $l_i(x)$ ($i = 0, 1, 2, \dots, n$) be the Lagrangian functions then show that $\sum_{i=0}^n l_i(x) = 1$. 10

- c) i) Describe power method for finding numerically largest eigen value of a square matrix.
- ii) Determine the largest Eigen value and the corresponding eigen vector of a system of equations associated with the following

matrix $\begin{bmatrix} 3 & 7 & 5 \\ 4 & 8 & 9 \\ 2 & 5 & 6 \end{bmatrix}$ 5+5=10

- d) Establish Gauss-Seidel iteration method for numerical solution of a system of n linear equations with n unknowns. Deduce the condition of convergence for this method.

$$6+4=10$$