

U.G. 1st Semester Examination - 2024**MATHEMATICS****[Skill Enhancement Course (SEC)]****Course Code : MATH-SEC-T-01****(Logic & Boolean Algebra)****[NEP-2020]**

Full Marks : 35

Time : $1\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Symbols have their usual meanings.*

1. Answer any five questions: $1 \times 5 = 5$
- a) Which of the following is well formed propositional formulas?
- (i) $\neg p \neg q$ (ii) $(p \wedge \neg q) \vee (q \rightarrow r)$
- b) Write the truth table of $p \rightarrow q$.
- c) Let's consider a following propositional language:
- p means "Mina is happy",
 q means "Mina paints a picture",
 r means "Rina is happy"
- Then formalize the following sentences: "if Mina is happy and paints a picture then Rina is not happy".

[Turn over]

- d) Prove that $p \vee T \Leftrightarrow T$.
- e) Let $X = \{1, 2, 3, 4, 5\}$. Is the relation $\Omega = \{(1, 1), (2, 2), (1, 2), (3, 3), (4, 4), (2, 1), (5, 5)\}$ a partially order on X ?
- f) If a set X has n number of elements, then what is the minimum number of elements contained in a partial order relation on X ?
- g) If a and b be two elements of a lattice $(L, \leq, \vee, \wedge)$ with $a \leq b$, then show that $a \wedge b = a$.
- h) Show that identity elements 0 and 1 are unique in a Boolean algebra.

2. Answer any two questions: $5 \times 2 = 10$

- a) Show that the propositions $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are equivalent.
- b) Let $n \in \mathbb{N}$ and $P_n = \{k \in \mathbb{N} : k \text{ divides } n\}$. Then show that $(P_n, \leq_n)$ is a partially order set, where the relation " $\leq_n$ " is defined by
 $a \leq_n b$ if and only if a divides b for all $a, b \in P_n$.
 Is $(P_n, \leq_n)$ a totally order set?
- c) Define a distributive lattice. For a positive integer n , let $D(n)$ be the set of all divisors of n . For elements $x, y \in D(n)$, define $x \leq y$ if and only if x divides y .
 Then show that $(D(n), \leq)$ is a distributive lattice, where $\vee = \text{lcm}$ and $\wedge = \text{gcd}$.

- d) Define the gates AND, OR and NOT and write their respective truth tables.

3. Answer any two questions: $10 \times 2 = 20$

- a) I) Construct the truth table of
 $((p \rightarrow q) \wedge (q \leftrightarrow r)) \vee \neg((q \rightarrow p) \wedge (p \leftrightarrow r))$.
- II) For a lattice $(L, \leq, \vee, \wedge)$ and $a, b, c \in L$, show that
- (i) $b \leq c \Rightarrow a \vee b \leq a \vee c$ and $a \wedge b \leq a \wedge c$
- (ii) $a \leq b, c \leq d \Rightarrow a \vee c \leq b \vee d$ and $a \wedge c \leq b \wedge d$
- (iii) $a \leq c \Leftrightarrow a \vee (b \wedge c) \leq (a \vee b) \wedge c$.

4+6

- b) I) What are the *inverse*, *converse* and "*contra-positive*" statements of a conditional statement? Write the converse and contra-positive statements of the statement
"If you do your homework, you will not be punished".
- II) What are the precedence rules of logical operators. Following the precedence rules, rewrite $p \wedge q \vee \neg r \leftrightarrow s$ using parentheses.

6+4

- c) I) Define a homomorphism between two lattices.
- II) Let $f: (\mathbb{N}, \leq) \rightarrow (\{0, 1, 2\}, \leq)$, where " $\leq$ " is the usual order is a homomorphism. If $f(m) = 0$ and $f(n) = 1$, then show that $m \leq n$.
- III) Define the direct product of two lattices. Show that the direct product of two distributive lattice is a distributive lattice.

2+3+5

- d) I) In Boolean algebra, prove or disprove
- $$(x_1'x_2'x_3'x_4') + (x_1'x_2'x_3'x_4) + (x_1'x_2'x_3x_4) + (x_1'x_2x_3x_4') = x_1'x_2'$$

- II) Formalize the following argument and verify its validity using truth table.

"If Dina won the competition, then either Rina came second or Sima came third. Sima did not come third. Thus, if Rina did not come second, then Dina did not win the competition."

5+5