

U.G. 4th Semester Examination - 2024

MATHEMATICS

[HONOURS]

Course Code : MATH-H-CC-T-10

(Linear Programming Problem & Game Theory)

Full Marks : 60

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Symbols and notations have their usual meanings.*1. Answer any ten questions: $2 \times 10 = 20$

- a) Show that a hyperplane is a convex set.
- b) Extreme points are finite in number. Justify.
- c) Prove that the set defined by $X = \{x : |x| \leq 12\}$ is a convex set.
- d) When are artificial variable used in an L.P.P.?
- e) Write down the dual of the following problem

Maximize $Z = 2x_1 + 3x_2 + x_3$ Subject to $2x_1 + x_2 - 3x_3 \leq 7$

$$2x_1 - x_2 + x_3 \leq 6$$

$$x_1 + 3x_2 + x_3 \leq 8$$

$$2x_1 + 3x_2 - x_3 \geq 12$$

$$x_1, x_3 \geq 0, x_2 \text{ is unrestricted in sign.}$$

[Turn Over]

- f) Find the basic feasible solutions of the equation

$$x_1 + 2x_3 = 1$$

$$x_2 + x_3 = 4$$

- g) Express the following LPP as standard maximization problem

$$\text{Minimize } Z = 4x_1 + 3x_2 + 2x_3$$

$$\text{Subject to } x_1 + 4x_2 - x_3 \leq 7$$

$$x_1 - 3x_2 + 2x_3 \leq 12$$

$$2x_1 + x_2 + x_3 = 8$$

$$4x_1 + 2x_2 - x_3 \geq 16$$

$$x_1, x_2, x_3 \geq 0$$

- h) State fundamental theorem of LPP.
- i) What is the criterion for the existence of multiple optimal solution for Transportation problem?
- j) How can you convert Unbalanced into a balanced Transportation Problem?
- k) Define a symmetric game. Why is it called so?
- l) State Fundamental theorem of Game Theory.
- m) Find out the extreme points (if any) of the convex set $S = \{(x, y) : |x| \leq 1, |y| \leq 2\}$.

- n) Is the point (1, 10) in the convex set of feasible solution determined by the constraints

$$2x_1 + 5x_2 \leq 40, x_1 + x_2 \leq 11, x_2 \geq 4, x_1, x_2 \geq 0.$$

- o) Give example of convex hulls in E^2 and E^3 .

2. Answer any four questions: 5×4=20

- a) Solve the following L.P.P by simplex method

$$\text{Maximize } Z = x_1 + x_2 + 3x_3$$

$$\text{Subject to } 3x_1 + 2x_2 + x_3 \leq 3$$

$$2x_1 + x_2 + 2x_3 \leq 2$$

$$x_1, x_2, x_3 \geq 0$$

- b) Solve the following L.P.P by graphical method

$$\text{Maximize } Z = 10x_1 + 15x_2$$

$$\text{Subject to } x_1 + x_2 \geq 2,$$

$$3x_1 + 2x_2 \leq 6,$$

$$x_1, x_2 \geq 0$$

- c) Show that the feasible solution $x_1=1, x_2=1, x_3=0$ and $x_4=2$ to the system

$$x_1 + x_2 + x_3 = 2$$

$$x_1 + x_2 - 3x_3 = 2$$

$$2x_1 + 4x_2 + 3x_3 - x_4 = 4$$

$$x_1, x_2, x_3, x_4 \geq 0$$

is not basic.

- d) For what value of a the game with the following payoff matrix is strictly determinable?

		A		
		I	II	III
B	I	a	5	2
	II	-1	a	-8
	III	-2	3	a

- e) State and prove weak duality theorem.
 f) Find out the optimal (minimum) assignment cost from the following cost matrix:

	I	II	III	IV
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

3. Answer any two questions:

10×2=20

- a) i) Prove that the set of all convex combinations of a finite number of points is a convex set.

- ii) Solve the following transportation problem: 4+6

	D ₁	D ₂	D ₃	
O ₁	5	1	8	12
O ₂	2	4	0	14
O ₃	3	6	7	4
	9	10	11	

- b) Solve the following L.P.P by using Big-M method

$$\text{Maximize } Z = 5x_1 - 2x_2 + 3x_3$$

$$\text{Subject to } 2x_1 + 2x_2 - x_3 \geq 2$$

$$3x_1 - 4x_2 \leq 3$$

$$x_2 + 3x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0.$$

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- c) Write down the dual of the following problem

$$\text{Minimize } Z = 3x_1 + 4x_2$$

$$\text{Subject to } x_1 + x_2 \leq 12$$

$$2x_1 + 3x_2 \leq 21$$

$$x_1 \leq 8$$

$$x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

Solve the dual problem, discuss the nature of the primal problem.

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- d) i) Prove that, the Transportation Problem always has a feasible solution.
- ii) Reduce the following game to 2×2 game and then solve it:

$$\begin{vmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{vmatrix}$$

4+6