

U.G. 3rd Semester Examination-2025**STATISTICS****[MAJOR]****Course Code : ST-M-T-3****(Linear Algebra and Numerical Analysis)****[NEP-2020]**

Full Marks : 50 (40+10)

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meaning.*1. Answer any **five** questions: 2×5=10a) Show that there cannot exist matrices A and

$$B \text{ such that } AB - BA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

b) Find the value of k so that the set $S = \{(1, 2, 1), (k, 3, 1), (2, k, 0)\}$ is linearly independent in R^3 .c) If A be a non-singular matrix, prove that $|A^{-1}| = |A|^{-1}$. If A is orthogonal then prove that $|A|^{-1} = |A|$.d) If 1, 6, and 8 are the eigenvalues of a 3×3 matrix A then find $|A|$ and $\text{tr}(A)$.*[Turn over]*

- e) Prove that $\Delta \log f(x) = \log \left[1 + \frac{\Delta f(x)}{f(x)} \right]$.
- f) If $f(x) = e^{ax}$, evaluate $\Delta^n f(x)$.
- g) Prove that $e^x = \left(\frac{\Delta^2}{E} \right) e^x \cdot \frac{Ee^x}{\Delta^2 e^x}$.
- h) Write down the relationship between absolute error and relative error.

2. Answer any **two** questions: 5×2=10

- a) Find the values of a and b so that the following system of equations has (i) a unique solution, (ii) infinitely many solutions, and (iii) no solution.

$$x + y + z = b$$

$$2x + y + 3z = b + 1$$

$$5x + 2y + az = b^2$$

- b) Use Gram-Schmidt process to obtain an orthogonal basis from the basis set $\{(1, 2, -2), (2, 0, 1), (1, 1, 0)\}$ of R^3 with the usual inner product.
- c) Find $f(2.5)$ using Newton's forward difference formula for the given data:

x	1	2	3	4	5	6
$f(x)$	0	1	8	27	64	125

- d) Derive Newton-Cotes formula from Lagrange's interpolation formula for numerical integration.

3. Answer any **two** questions: 10×2=20

- a) i) Show that the eigenvectors corresponding to two distinct eigenvalues of a real symmetric matrix are orthogonal.
- ii) Let $\mathbf{X}$ be an eigenvector of an $n \times n$ matrix A associated with the eigenvalue a . Prove that $P^{-1}\mathbf{X}$ is an eigenvector of the matrix $P^{-1}AP$ associated with a .

5+5

- b) i) Extend the set $\{(1, 1, 1, 1), (1, -1, 1, -1)\}$ to a basis of R^4 .
- ii) Check the definiteness of the quadratic form $x^2 + y^2 + z^2 + 3xz$.
- c) Describe bisection method for computing the real roots of the equation $f(x) = 0$. Find a real root of the equation

$$x^3 + x - 5 = 0$$

by this method correct to two decimal places. 5+5

- d) i) Solve the following system by Gauss elimination method

$$x - 2y + 9z = 8$$

$$3x + y - z = 3$$

$$2x - 8y + z = -5.$$

- ii) Apply Lagrange's interpolation formula to find $f(x)$, given
 $f(1) = 2, f(2) = 4, f(3) = 8, f(4) = 16$
and $f(7) = 128$. 5+5

[Internal Assessment: 10]
