

U.G. 2nd Semester Examination - 2022

STATISTICS

[HONOURS]

Course Code : STAT-H-CC-T-4

(Probability and Probability Distributions-II)

Full Marks : 50(40+10)

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meaning.*

1. Answer any **five** questions: $2 \times 5 = 10$
- Define random variable. Explain how it differs from an empirical variable.
 - Write down the properties of cdf in the context of discrete random variable.
 - Define mathematical expectation.
 - State the loss of memory property of a distribution. Name a distribution that satisfies this property.
 - Let (X, Y) follow bivariate normal $(-1, 2, 4, 5, 1/3)$. Find $\text{Var}[E(Y|X)]$.

- What do you mean by point of inflection? Give an example.
- If $X \sim N(0, 1)$, find $E(\sin(x))$.
- State two properties of a bivariate normal distribution.

2. Answer any **two** questions: $5 \times 2 = 10$

- Let $X \sim N(\mu, \sigma^2)$. Find the mgf of $Y = X^2$.
- Let X be an integer valued random variable with p.g.f. $P(s)$. Let a and b be non-negative integers and write $Y = aX + b$. Find the pgf of Y .
- Write down the pdf of Pareto distribution. Derive a condition under which the moment of order n exists.
- Let X and Y be two independent Geometric

(p) random variables. Find $E\left(\frac{X^2 + Y^2}{XY}\right)$.

3. Answer any **two** questions: $10 \times 2 = 20$

- Let X be a non-negative continuous random variable with r^{th} order raw moment given by $(r+1)!2^r$. Derive the skewness of X and comment. 5

- ii) Let X and Y be two independent $U(0, 1)$ random variables and let $Z = \max(X, Y)$ and $W = \min(X, Y)$. Find the covariance between Z and W . 5

- b) i) Let (X, Y) be jointly distributed with pdf

$$f(x, y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}.$$

Find the marginal pdfs of X and Y and the conditional distribution of Y given $X=x$.

5

- ii) Derive the mode of Poisson distribution with parameter λ . 5

- c) i) Find the expressions for the mean and median of log-normal distribution. 5

- ii) Write down the pmf of negative binomial distribution with index r and parameter p . Discuss how it is related to geometric distribution. 5

- d) i) Suppose X follows Cauchy $(0, 1)$ distribution. Show that $P\left(\frac{X}{1+X^2} < 0\right) = \frac{1}{2}$.

Hence or otherwise find $E\left(\frac{X}{1+X^2}\right)$. 5

- ii) Show that for a Poisson distribution with unit mean, mean deviation about mean is $2/e$ times the standard deviation. 5

[Internal Assessment: 10]
