

U.G. 6th Semester Examination - 2025

MATHEMATICS**[HONOURS]****Course Code : MATH-H-CC-T-14****(Probability and Statistics)****[New Syllabus]**

Full Marks : 60

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**The symbols and notations have their usual meanings.*

1. Answer any ten questions: $2 \times 10 = 20$
- a) If $X^* = (X - \mu)/\sigma$ is a standardized random variable, prove that (i) $E(X^*) = 0$, (ii) $Var(X^*) = 1$.
 - b) Define conditional expectation.
 - c) Find the probability of drawing two kings at random from a deck of 52 ordinary cards if the cards are (i) not replaced and (ii) replaced.
 - d) Define coefficient of skewness and kurtosis of a distribution.
 - e) The quantity $E[(X - m)^2]$ is minimum when $m = \mu = E(X)$.

[Turn Over]

- f) Prove that $-1 \leq \rho \leq 1$, where ρ is the correlation coefficient.
- g) Find the mathematical expectation of a discrete random variable X whose probability function is given by $f(x) = (1/4)^x$, $(x = 1, 2, 3, \dots)$
- h) Find the probability of not getting 8 or 9 total on either of two tosses of a pair of fair dice.
- i) A shelf has six chemistry books and 4 mathematics books. Find the probability that 3 particular mathematics books will be together.
- j) Find i) the covariance, ii) correlation coefficient of two random variables X_1 and X_2 if $E(X_1) = 2$, $E(X_2) = 3$, $E(X_1 X_2) = 10$, $E(X_1^2) = 9$, $E(X_2^2) = 16$.
- k) The joint density function of two continuous random variables X and Y is

$$f(x, y) = \begin{cases} kxy & 0 < x < 4, 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of the constant k .

- l) Let X have density function

$$g(x) = \begin{cases} 1/(a_2 - a_1) & a_1 \leq x \leq a_2 \\ 0 & \text{otherwise} \end{cases}$$

Find the k^{th} moment about i) the origin, ii) the mean.

- m) Find the characteristic function of a random variable X having density function

$f_1(x) = ke^{-r|x|}$, $-\infty < x < \infty$, where $r > 0$ and k is suitable constant.

- n) Find the probability that in successive tosses of a fair die, a 3 will come up for the first time on the 4th toss.
- o) State Chebyshev's inequality for a continuous random variable.
2. Answer any four questions: 5 × 4 = 20
- a) If X and Y are independent random variables, then show that $E(XY) = E(X)E(Y)$. 5
- b) Define type-I and type-II errors.

The probability density function of the random variable X is

$$f(x) = \begin{cases} \frac{1}{l} e^{-x/l} & x > 0 \\ 0 & x \leq 0 \end{cases}, \text{ where } l > 0. \text{ For testing}$$

the hypothesis $H_0: l = 3$ against $H_A: l = 5$, a test is given as "Reject H_0 if $X \geq 4.5$ ". Find the probability of type-I error and power of the test. 2+3

- c) Find the variance and standard deviation of the sum obtained by tossing a pair of fair dice. 5
- d) Show that the mean and variance of the uniform distribution

$$f(x) = \begin{cases} \frac{1}{d-c} & c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

are given respectively by (i) $\mu = \frac{1}{2}(c+d)$,

(ii) $\sigma^2 = \frac{1}{12}(d-c)^2$. 5

e) State and prove law of large numbers theorem. 5

f) The standard deviation of the lifetimes of a sample of 200 electric light bulbs was computed to be 100 hours. Find (a) 95%, (b) 99% confidence limits for the standard deviation of all such electric light bulbs. 5

3. Answer any two questions: 10×2=20

a) Design a decision rule to test the hypothesis that a coin is fair if a sample of 64 tosses of the coin is taken and if a level of significance of (a) 0.05, (b) 0.01 is used. 5+5

b) The election returns showed that a certain candidate received 46% of the votes. Determine the probability that a poll of (a) 200, (b) 1000 people selected at random from the voting population would have shown a majority of votes in favor of the candidate. 5+5

c) If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.002, determine the probability that out of 3500 individuals, (i) exactly 4, (ii) more than 3, individuals will suffer a bad reaction. 5+5