

U.G. 6th Semester Examination - 2022**STATISTICS****[PROGRAMME]****Discipline Specific Elective (DSE)****Course Code : STAT-G-DSE-T-1B&1D**

Full Marks : 50(40+10)

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meanings.***Answer all the questions from selected Option.****OPTION-A****STAT-G-DSE-T-1B****(Survival Analysis)**

1. Answer any **five** questions: $2 \times 5 = 10$
- What is progressive censoring? Explain with an example.
 - What is median survival time?
 - Define DFR. Cite an example where we can observe this.
 - What do you mean by Duration of an epidemic?

- Find out the inter relationship between survival function and hazard function.
- What is the difference between death density function and hazard function?
- What is the advantage of hazard functions over survival functions?
- Cite an example of each of type I and type II censoring in the context of biology.

2. Answer any **two** questions: $5 \times 2 = 10$
- Find out the hazard function for a time to event variable which follows Rayleigh distribution. Discuss the shape of the hazard functions with different parameter choices.
 - Write a note on minimum Chi-square methods in the context of competing risk theory.
 - Show that the Exponential distribution is characterized by its constant hazard.
 - Derive the expression for the mean residual life for a time to event variable which follows Weibull distribution.
3. Answer any **two** questions: $10 \times 2 = 20$
- Derive the Actuarial estimator for a group of censored failure time data. 7
 - Discuss how you would obtain the estimator

[Turn over]

of cumulative hazard function using the above estimator. 3

- b) i) Obtain the variance of the estimated survival function, derived using Actuarial method. 7
- ii) Discuss the difference between Kaplan Meier method and Actuarial method. 3
- c) i) Discuss the bathtub shaped failure rate life distributions. 3
- ii) Suppose the time to event, T , is a random variable following Gamma distribution. Find out the range(s) of the parameters of the distribution for which $h(t)$, the hazard at t , is (I) increasing, (II) decreasing and (III) constant. 7
- d) i) Discuss how you would derive the hazard curves under a competing risk model. 5
- ii) What do you mean by dependent and independent risk? 3
- iii) Citing a real life example, discuss the Bivariate normal dependent risk model. 2

[Internal Assessment: 10]

OPTION-B
STAT-G-DSE-T-1D
(Actuarial Statistics)

1. Answer any **five** questions: 2×5=10
- a) Express e_x in terms of l_x .
- b) Explain the term 'effective rate of discount'.
- c) Explain the following concepts:
- i) Insured
- ii) Speculative risk
- d) Obtain accumulated value of Rs. 50,000 at the end of 5th year with effective rate of interest 7%.
- e) State any two characteristics of insurable risk.
- f) If Rs. 1,00,000 is invested on 1st January 2018 and the force of interest per annum is 0.05, obtain the accumulated value on 1st January 2019.
- g) Explain the role of an actuary in an insurance company.
- h) A person has invested Rs.5,000 in a savings scheme. After 10 years he is entitled to receive Rs.7,500. What rate of interest is realized in the transaction?
2. Answer any **two** questions: 5×2=10
- a) A loan of Rs. 10,00,000 is taken and it has to be

repaid in 10 equal installments payable yearly at the beginning of the year. Based on 7% annual effective rate of interest determine the amount of installments.

- b) The survival rates (p_x) for a certain animal population are as follows:

Age (in months)							
x	0	1	2	3	4	5	6
p_x	0.8	0.75	0.7	0.4	0.2	0.1	0

For a radix of 100000 obtain the columns l_x , L_x , and T_x .

- c) Show that the condition for mutually advantageous policy is $G \geq \mu$, where G is onetime premium and μ is expected value of loss random variable. State the assumptions.

- d) Let X have a discrete distribution as follows:

$$P(X=100)=0.6, P(X=175)=0.3, P(X=200)=0.1.$$

If the insurer effects excess of loss reinsurance with retention level 150, what is the distribution of the non-zero payments made by the reinsurer?

3. Answer any **two** questions: $10 \times 2 = 20$

- a) (i) State any two properties of survival function $S(x)$. Derive the expression for $S(x)$ in terms

of constant force of mortality. 4

- ii) An individual is facing a random loss, X , which is uniformly distributed over $(0, 200)$. The individual can purchase partial insurance cover under which the insurer will pay $\max(0, X - 20)$, and the premium for this cover is 85. The individual has wealth 250 and makes decisions on the basis of the utility function $u(x) = x^{2/3}$ for $x > 0$.

(I) Show that the individual is risk averse.

(II) Will the individual purchase insurance cover? 6

- b) i) Suppose the life length random variable is modeled by a distribution with force of mortality as specified below:

$$\mu_s = \begin{cases} 0.01 & \text{if } 0 < s < 15 \\ 0.02 & \text{if } 15 \leq s \leq 25 \\ 0.03 & \text{if } s \geq 25 \end{cases}$$

Find corresponding $S(x)$, $f(x)$ and probability density function $g(t)$ of $T(20)$. 5

- ii) If $\mu_x = 0.02$ for $40 \leq x \leq 47$ evaluate ${}_2p_{43}$ and ${}_3/4q_{40}$. 5

c) i) Explain the term 'Annuity' with an illustration.

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ii) Explain Annuity certain due and Annuity certain immediate.

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iii) An insurer is considering offering complete insurance cover against a random loss, X , where $E[X] = V[X] = 100$ and $P(X > 0) = 1$. The insurer adopts the utility function $u(x) = x - 0.001x^2$ for decision making purposes. Calculate the minimum premium that the insurer would accept for this insurance cover when the insurer's wealth, W , is (A) 100, (B) 200 and (C) 300.

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d) i) Obtain $E(Z)$, where Z is net single premium in terms of V for

A) n -year term insurance

B) whole life insurance

C) n -year pure endowment insurance.

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ii) If for the annuity certain the payments are made regularly at the beginning of the year, derive that

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$$\ddot{S}_{n|} = (1+i)^n \ddot{a}_{n|}$$

[Internal Assessment: 10]