

## U.G. 1st Semester Examination - 2025

## MATHEMATICS

[Skill Enhancement Course (SEC)]

Course Code : MATH-SEC-T-01

(Logic &amp; Boolean Algebra)

[NEP-2020]

Full Marks : 35

Time :  $1\frac{1}{2}$  Hours*The figures in the right-hand margin indicate marks.**Symbols / Notations have their usual meanings.*

1. Answer any **five** questions:  $1 \times 5 = 5$
- a) If  $P(x, y)$  is " $x^2 + y = 12$ ", then find the truth value of  $P(2, 8)$  and  $P(1, 6)$ .
  - b) Define negative proposition ( $\neg p$ ) with its truth table.
  - c) Prove that  $\neg\neg p \Leftrightarrow p$ .
  - d) Give an example of a distributive lattice.
  - e) If  $(L_1, \leq_1, \vee_1, \wedge_1)$  and  $(L_2, \leq_2, \vee_2, \wedge_2)$  are two lattices, how do you define *meet* ( $\wedge$ ) and *join* ( $\vee$ ) for the product lattice  $L = L_1 \times L_2$ ?
  - f) In Boolean algebra  $(B, +, 0, 1, ')$ , show that  $1' = 0$  and  $0' = 1$ .

*[Turn over]*

g) Is the Boolean function  $f(x, y, z) = (x \cdot y' + y \cdot z)' + x \cdot z$  in Disjunctive Normal Form (D.N.F.)?

h) Differentiate between OR gate and AND gate.

2. Answer any two questions:  $5 \times 2 = 10$

a) Using Precedence rules of logical operators, insert the parentheses in the compound expression  $p \rightarrow q \vee r \wedge \neg p \leftrightarrow \neg r$ . Write its truth table.  $2+3$

b) Define binary compositions in the power set  $P(X)$  of a nonempty set  $X$  to make  $P(X)$  a Boolean algebra with justification.  $5$

c) In Boolean algebra  $(B, +, 0, 1, ')$ , prove the associative law:

$$a + (b + c) = (a + b) + c$$

and then using duality principle of Boolean algebra, establish

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c. \quad 4+1$$

d) In a lattice  $(L, \leq, \vee, \wedge)$ , prove that

$$a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c). \quad 5$$

3. Answer any two questions:  $10 \times 2 = 20$

a) i) Consider the argument:

*"If I am guilty, then I must be punished: I must not be punished. Thus, I am guilty."*

Express the argument in a compound proposition. Is the argument logically correct?

ii) Simplify the following Boolean expression and then draw a circuit for it:

$$(x \cdot y + z') \cdot (y' + x' \cdot z) + x' \quad (2+3)+5$$

b) i) What is the "duality principle" of propositions. Find dual of

$$(p \wedge q) \vee p$$

and using duality principle, show that

$$(p \wedge q) \vee p \equiv p.$$

ii) Without using truth table, find the Full Disjunctive Normal Form (F.D.N.F.) of the Boolean function

$$f(x, y, z) = x + (x' \cdot y' + x' \cdot z)'. \quad (4+1)+5$$

c) i) In a distributive lattice  $(L, \leq, \vee, \wedge)$ , prove that  $a \vee (a \wedge b) = a$

and then

$$a \vee b = a \vee c, a \wedge b = a \wedge c \Rightarrow b = c.$$

- ii) Convert the *Full Disjunctive Normal Form (F.D.N.F)*

$$f(x,y,z) = x \cdot y' \cdot z' + x' \cdot y \cdot z' + x' \cdot y' \cdot z + x \cdot y \cdot z$$

to *Full Conjunctive Normal Form (F.C.N.F)*. (3+3)+4

- d) i) In a lattice  $(L, \leq, \vee, \wedge)$ , prove that  $a \leq b \Leftrightarrow a \wedge b = a \Leftrightarrow a \vee b$ .
- ii) Show that the compound proposition  $(p \wedge q) \vee p \Leftrightarrow \neg p$  is a contradiction.

5+5