

U.G. 3rd Semester Examination - 2020**STATISTICS****[HONOURS]****Course Code: STAT-H-CC-T-5****(Linear Algebra and Numerical Analysis)**Full Marks : 75 (60+15) Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meaning.*

1. Answer any **ten** questions: $2 \times 10 = 20$
- Prove that the diagonal elements of a real skew symmetric matrix are 0.
 - If A is an idempotent matrix prove that (I-A) is also an idempotent matrix.
 - Prove that if A and B are orthogonal matrices of same order then AB is also orthogonal.

- d) Show that there cannot exist matrices A and

$$B \text{ such that } AB - BA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

- e) Evaluate: $\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}.$

- f) Find the value of k so that the set $S = \{(1, 2, 1), (k, 3, 1), (2, k, 0)\}$ is linearly independent in R^3 .
- g) If A be a non-singular matrix, prove that $|A^{-1}| = |A|^{-1}$. If A is orthogonal then prove that $|A|^{-1} = |A|$.
- h) If A is a symmetric matrix of order m and P be an m x n matrix, prove that $P'AP$ is a symmetric matrix. Check whether $P'AP$ is skew symmetric or not, assume A is skew symmetric matrix.
- i) If 1, 6, and 8 are the eigenvalues of a 3×3 matrix A then find $|A|$ and $\text{tr}(A)$.
- j) Prove that $(1, 1)$ is an eigenvector of the matrix $\begin{pmatrix} 1 & -2 \\ -5 & 4 \end{pmatrix}.$

k) Evaluate: $\Delta^2(3e^x)$.

l) Prove that $\Delta \log f(x) = \log \left[1 + \frac{\Delta f(x)}{f(x)} \right]$.

m) If $f(x) = e^{ax}$, evaluate $\Delta^n f(x)$.

n) Prove that $e^x = \left(\frac{\Delta^2}{E} \right) e^x \cdot \frac{E e^x}{\Delta^2 e^x}$.

o) Write down the relationship between absolute error and relative error.

2. Answer any **four** questions: $5 \times 4 = 20$

a) Find the values of a and b so that the following system of equations has (i) a unique solution, (ii) infinitely many solutions, and (iii) no solution:

$$x + y + z = b$$

$$2x + y + 3z = b + 1$$

$$5x + 2y + az = b^2$$

b) Let I be the 4×4 identity matrix and $v = (1, 2, 3, 4)^t$, where t denotes the transpose. Find the determinant of $(I + vv^t)$.

c) Prove that the set $S = \{(2, 1, 1), (1, 2, 1), (1, 1, 2)\}$ is a basis for R^3 .

d) Reduce the quadratic form $5x^2 + y^2 + 10z^2 - 4yz - 10zx$ to the normal form and show that it is positive definite.

e) Find $f(2.5)$ using Newton's forward difference formula for the given data:

x	1	2	3	4	5	6
f(x)	0	1	8	27	64	125

f) Derive Newton-Cotes formula from Lagrange's interpolation formula for numerical integration.

3. Answer any **two** questions: $10 \times 2 = 20$

a) i) Show that the eigenvectors corresponding to two distinct eigenvalues of a real symmetric matrix are orthogonal.

ii) Let X be an eigenvector of an $n \times n$ matrix A associated with an eigenvalue a. Prove that $P^{-1}X$ is an eigenvector of the matrix $P^{-1}AP$ associated with a.

b) i) Use Gram-Schmidt process to obtain an orthogonal basis from the basis set $\{(1, 2, -2), (2, 0, 1), (1, 1, 0)\}$ of the Euclidean space R^3 with standard inner product.

- ii) Extend the set $\{(1, 1, 1, 1), (1, -1, 1, -1)\}$ to a basis of $\mathbb{R}^4$.
- c) From Newton-Cotes formula derive trapezoidal rule. Hence calculate the value of
$$\int_0^{\pi/2} e^{\cos x} dx.$$
- d) Describe Newton-Raphson Method. Using Newton-Raphson Method find the positive root of $x = e^x = 0$ correct to three decimal places.

[Internal Assessment: 15]
