

U.G. 5th Semester Examination - 2024**PHYSICS****[HONOURS]****Course Code : PHY-H-CC-T-12****(Statistical Mechanics)****[New Syllabus]****Full Marks : 40****Time : $2\frac{1}{2}$ Hours***The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.***GROUP-A****1. Answer any five questions: $2 \times 5 = 10$**

- a) Define the concept of microstates and macrostates in statistical mechanics.
- b) State the postulate of equal a priori probability.
- c) What is λ transition in liquid Helium?
- d) What is the Boltzmann entropy formula? Write its expression.
- e) Explain the concept of the partition function.

[Turn over]

- f) Show that for a canonical system, mean energy is given by -

$$\langle E \rangle = -\frac{\partial}{\partial \beta} (\ln Z)$$

Where Z is canonical partition function.

- g) How many ways can 5 Bosons be arranged in three quantum states?
h) Write the expression for Bose-Einstein distribution and mention one application.

GROUP-B

2. Answer any two questions: $5 \times 2 = 10$

- a) Discuss the differences between Bose-Einstein, Fermi-Dirac, and Maxwell-Boltzmann statistics. Provide examples of systems where each applies. What is meant by Fermi-level?

$2+1+2$

- b) What is the phase trajectory of a simple pendulum performing small oscillations? Show that the area enclosed by the trajectory is equal to the product of the total energy E and the time period T of the pendulum.

$2+3$

- c) A system consists of three distinguishable particles with energy levels $0, \epsilon, 2\epsilon$. Calculate the partition function, entropy and the average energy at temperature T .

$3+1+1$

- d) Find the variation of the specific heat C_v as a function of the temperature T for photon gas confined in 1D box. 5

GROUP-C

Answer any two questions:

$10 \times 2 = 20$

3. a) Consider a photon gas confined in a volume V at temperature T . Show that the number of photons in this volume is proportional to T^3 .
b) A photon gas is confined in volume V at temperature T . If the volume is increased adiabatically to $2V$, determine the final temperature.
c) Derive Wein's displacement law from Planck's law. $3+2+5$
4. a) A particle in 1D has energy

$$E = \frac{p^2}{2m} + \lambda q^4$$

where q and p are the generalized coordinate and momentum respectively. Show that heat capacity of a gas comprising of N such particles is

$$C_v = \frac{3}{4} Nk$$

- b) The grand canonical partition function for an ideal gas is given by

$$Z_G(T, V, \mu) = \exp \left[e^{\mu/kT} \frac{V}{\lambda^3} \right]$$

where $\lambda = \frac{h}{\sqrt{2\pi m k T}}$ is the thermal de Broglie wavelength. Calculate average number of particles and hence calculate the equation of state of the system.

- c) Explain why microcanonical canonical and grand canonical ensembles give almost same results for a system with large number of particles? 4+3+3
- 5. a) Define Bose-Einstein condensation and explain the conditions under which it occurs.
- b) Derive the critical temperature T_C for Bose-Einstein condensation for a system of non-interacting bosons in three dimensions.
- c) Calculate T_C for 10^{20} bosons in a volume of 1cm^3 . 3+4+3
- 6. a) Sketch the Fermi-Dirac distribution function and its derivative for $T = 0\text{ K}$ and $T > 0\text{ K}$ showing clearly the Fermi energy.
- b) Explain physically how the electronic specific heat of a metal behaves as a function of temperature.
- c) Deduce the pressure-volume relationship for a free electron gas obeying Fermi-Dirac statistics at 0K . Hence find an expression for the bulk modulus of the gas. 3+3+4