

U.G. 6th Semester Examination-2025

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-13

(Electro-Magnetic Theory)

[New Syllabus]

Full Marks : 40

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.*

(Treat all bold letters as a vector)

1. Answer any five questions: $2 \times 5 = 10$

- a) Define the single mode and multi mode propagation in optical fibre.
- b) Define the specific rotation and write down the relation between angle of rotation and specific rotation.
- c) Define the *Electric displacement*(**D**). Write down the Maxwell's equation associated with the electric displacement (**D**).
- d) The specific rotation of quartz at $5086 \text{ \AA}$ is 29.73 deg/mm . Calculate the difference in the refractive indices.

[Turn Over]

- e) For an electromagnetic wave propagating in the z direction, let the x and y components of the electric field be given by $E_x = E_{x0} \cos(\omega t - kz)$ and $E_y = E_{y0} \cos(\omega t - kz + \theta)$.

Write down the conditions so that they can form circularly polarized and elliptical polarized wave form.

- f) Write down the electric field (**E**) and magnetic field (**B**) in terms of scalar potential (Φ) and vector potential (**A**). Show that magnetic field (**B**) remains unchanged under this gauge transformation $A' = A + \nabla \lambda$ where λ is some scalar function.

- g) For a light wave, the relation between angular frequency (ω) and the wave vector (**k**) is given by $\omega(k) = \frac{ck}{n(k)}$ where c , $n(k)$ are the velocity of light and refractive index respectively. Show that the group velocity (v_g) can be written as
- $$v_g = \frac{c}{n(\omega) + \omega \frac{dn}{d\omega}}$$

- h) Write down the *advantage* and *disadvantage* of Coulomb gauge transformation in electromagnetic theory.

2. Answer any two questions:

5×2=10

- a) Define the *Numerical Aperture (NA)*. Show that the *Numerical Aperture (NA)* of optical fiber is given by $NA = 1/n_0 \sqrt{n_1^2 - n_2^2}$ where n_0 , n_1 and n_2 are the refractive indices of air, core and cladding of the optical fiber respectively.

2+3

- b) Write down the Fresnel theory of optical rotation. Calculate the angle of rotation.

2+3

- c) In polarized medium, derive these two relations: $D = E + \epsilon_0 P$ and $H = \frac{1}{\mu_0} B - M$ and write down four Maxwell's equation. 2+2+1

- d) Consider two plane wave fields of the form $E = \epsilon_1 E_0 e^{i(k \cdot x - \omega t)}$ and $B = \epsilon_2 B_0 e^{i(k \cdot x - \omega t)}$.

For a non-conducting medium show that the energy flow is given by $S = \frac{c}{8\pi} \sqrt{\frac{\epsilon}{\mu}} E_0^2 \epsilon_3$ where ϵ_3 is a unit vector in the direction of **k** and ϵ_1, ϵ_2 are the unit vector along the axes "1" and "2" respectively.

5

3. Answer any two questions: $10 \times 2 = 20$

a) i) State the Poynting's theorem. Prove the differential continuity equation $\frac{\partial u}{\partial t} + \nabla \cdot S = -J \cdot E$, where u , S are represent the total energy density and Poynting vector respectively.

ii) A charged parallel capacitor is placed (with a uniform electric field $E = E_0 \hat{z}$) in a uniform magnetic field $B = B_0 \hat{x}$. Find the electromagnetic momentum in the space between the plates. $2+5+3$

b) i) For a conducting medium, write down the Maxwell's equation. Show that the electromagnetic wave vector (k) is complex and is given by $k^2 = \mu \epsilon \omega^2 + i \mu \sigma \omega$, where σ is the conductivity of the medium and other symbols have meaning as in electromagnetic theory. Find out the form of plane wave for both electric field (E) and magnetic field (B).

ii) Show that for poor conductor the attenuation factor is independent of frequency. Define the skin depth. Relate the skin depth with the attenuation factor.

$1+3+3+2+1$

c) i) Discuss the construction and working principle of Nicole prism. How can one use Nicole prism as analyzer?

ii) A sugar solution in a tube of length 20 cm produces optical rotation of 13° . The solution is diluted to one-third of its previous concentration. Find optical rotation produced by 30cm long tube containing the diluted solution. $5+2+3$

d) i) The motion of electron inside the non-conducting material is like a damped oscillator in the presence of external electric field. Show that the permittivity of the medium is frequency dependent and corresponding dielectric constant is given by

$$\epsilon_r = 1 + \frac{Nq^2}{m\epsilon_0} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 + i\gamma_j \omega}$$

where ω_j is frequency of j^{th} type oscillator (electron) γ_j is the corresponding damping factor and f_j is the number of such electrons per atom.

- ii) In this dispersive medium, prove that the refractive index is given by

$$n(\omega) \approx 1 + \frac{Nq^2}{2m\epsilon_0} \sum \frac{f_j(\omega_j^2 - \omega^2)}{(\omega_j^2 - \omega^2)^2 + \gamma_j^2 \omega^2}$$

6+4