

U.G. 3rd Semester Examination - 2021

STATISTICS

[HONOURS]

Course Code: STAT-H-CC-T-5

(Linear Algebra and Numerical Analysis)

Full Marks : 75 (60+15)

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meaning.*1. Answer any **ten** questions: $2 \times 10 = 20$

- Check whether the set of vectors $\{(1, 2, 1), (2, 0, -1), (4, 4, 0)\}$ is a spanning set for $\mathbb{R}^3$.
- Check whether the vectors $u=(1,2,1,3)$, $v=(3,1,2,1)$ are linearly independent or not.
- prove that any matrix can be uniquely expressed as the sum of a symmetric matrix and a skew symmetric matrix.

- If A and B are two matrices such that $AB=A$ and $BA=B$, then prove that A^T and B^T are idempotent.

- Find the values of a and b so that the matrix.

$$A = \begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & a & b \end{pmatrix} \text{ is of rank 2.}$$

- Let $A = \begin{vmatrix} 1+x & 2 & 2 \\ 2 & 1+x & 2 \\ 2 & 2 & 1+x \end{vmatrix}$. Find the

determinant of A. Find the value of x for which the rank of the matrix A is less than 3.

- If λ be an eigenvalue of a non-singular matrix

A, show that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1} .

- Define consistency of a linear system of equations. Show that a linear homogeneous system of equations is always consistent.
- If 1, 5 and 6 are eigenvalues of a 3×3 matrix A, what are the eigenvalues of A^T and A^{-1} ?
- In $\mathbb{R}^3$, $\alpha = (4, 3, 5)$, $\beta = (0, 1, 3)$, $\gamma = (2, 1, 1)$. Examine if α is a linear combination of β and γ .

- k) If the absolute error is 0.005 and relative error is 3.264×10^{-6} , then find the true value and percentage error.
- l) State with usual notation the Newton-Raphson formula.
- m) Distinguish between interpolation and least squares method.
- n) How many fourth-degree polynomials pass through the points (1, 1), (2, 8), (3, 27) and (4, 64)?
- o) Consider the interpolating polynomial for $f(x) = x^3$ based on the points (1, 1), (2, 8), (3, 27), and (4, 64). Find an upper bound for the interpolation error on [1, 4].

2. Answer any **four** questions: $5 \times 4 = 20$

- a) Let X and Y be two subspaces of a vector space V.
 - i) Prove that the intersection $X \cap Y$ is also a subspace of V.
 - ii) Show that the union $X \cup Y$ need not be a subspace of V.

- b) Determine the dimension as well as a basis for the space spanned by

$$S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix} \right\}$$

- c) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{pmatrix}. \text{ Hence compute } A^{-1}.$$

- d) Diagonalize the symmetric matrix

$$A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}.$$

- e) Using bisection method, find the root of $x^3 - 3x - 5 = 0$ correct to three significant figures.
- f) Apply Lagrange's interpolation formula to find $f(x)$, if $f(1)=2, f(2)=4, f(3)=8, f(4)=16$ and $f(7)=128$.

3. Answer any **two** questions: $10 \times 2 = 20$

- a) If S be a real skew symmetric matrix of order n
- i) Prove that the matrix $S - I_n$ is non-singular.
 - ii) If X be an eigenvector of S with eigenvalue a then prove that X is also an eigenvector of the matrix $(S - I_n)^{-1}(S + I_n)$ with eigenvalue $\frac{a+1}{a-1}$.
- b) i) Show that a real symmetric matrix is positive definite if and only if all its eigenvalues are positive.
- ii) Check the definiteness of the quadratic form $x^2 + y^2 + z^2 + 3xz$.
- c) Describe fixed point iteration method. Using this method, find the root of $2x - \log_{10} x - 7 = 10$ correct up to four decimal places.
- d) Derive Newton-Cotes formula from Lagrange's interpolation formula for numerical integration.

[Internal Assessment: 15]