

U.G. 2nd Semester Examination - 2022**STATISTICS****[HONOURS]****Generic Elective Course (GE)****Course Code : STAT-H-GE-T-2****(Introductory Probability)**

Full Marks : 50(40+10) Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer any **five** questions: 2×5=10
 - a) Define 'event' with an example.
 - b) One big coin, one small coin and a die are thrown simultaneously. Write down the sample space.
 - c) "The expectation of a random variable does not always exist"—Justify with an example.
 - d) State DeMoivre-Laplace Central Limit Theorem.
 - e) Write down the p.m.f. of a discrete distribution having the 'loss of memory property'.
 - f) Define conditional probability.

[Turn Over]

- g) Give the empirical definition of probability.
- h) If X is distributed uniformly over $[-10, 4]$, what is the probability of observing an X that is greater than -1 ?

2. Answer any **two** questions: 5×2=10

- a) State and the law of compound probability for three events. A shipment of 15 water heaters arrived in a shop was checked and it was found that 4 are with defective thermostats. Two water heaters will be selected at random one after another. Find the probability that (i) both are defective, (ii) the second one is defective.
- b) Write down the p.d.f. of a beta distribution with parameters m and n, clearly mentioning the ranges of these parameters. Derive the mean and variance of this distribution.
- c) Verify whether weak law of large numbers holds for the following sequence of independent random variables $\{X_n, n=1, 2, \dots\}$:

$$P\{X_n=1/\sqrt{n}\}=2/3, P\{X_n=-1/\sqrt{n}\}=1/3.$$
- d) Obtain the constant 'a' for which the following function would be a p.m.f. of a discrete random variable X.

$f(x) = a(1+x), x=2, 3, \dots, n.$ Find $P(X>2)$.

3. Answer any **two** questions: $10 \times 2 = 20$

a) Define m.g.f. of a random variable. Derive the m.g.f. of a random variable which is normally distributed with mean μ and variance σ^2 . Hence find μ_3 .

b) i) In a large municipal town, 75% of the residents are vaccinated against a disease and 54% are attacked by that disease. Find a lower bound of the percentage of vaccinated people who have been attacked.

ii) Define negative binomial distribution and mention an application of it. Find the mean of this distribution. $4+6$

c) Suppose $A_1, A_2, \dots, A_n$ are n events defined on a sample space and they are not necessarily mutually exclusive. Derive an expression for the probability of occurrence of at least one of them in terms of the individual probabilities and probabilities of their joint occurrences.

d) Write notes on:

i) Bayes theorem

ii) Chebyshev's inequality $5+5$

[Internal Assessment: 10]
