

U.G. 6th Semester Examination - 2025

MATHEMATICS**[HONOURS]**

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-03B

(Biomathematics)

[New Syllabus]

Full Marks : 60

Time : $2\frac{1}{2}$ Hours

*The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in
their own words as far as practicable.
The symbols and notations have their usual meanings.*

1. Answer any ten questions:

 $2 \times 10 = 20$

- a) What assumptions are made by Malthus for formulating the population growth model?
- b) What is meant by bifurcation?
- c) Define the term 'supercritical Hopf'.
- d) Define basic reproduction number.
- e) Explain a continuous age-structured model.
- f) For the predator-prey model: $\frac{dx}{dt} = rx(1-y)$,

$\frac{dy}{dt} = my(1-x)$ (r and m are positive parameters), determine whether the system is stable at (0.0).

- g) In an ecosystem that follows the logistic growth model, the initial population is 900 individuals. The growth rate constant is 0.1, and the carrying

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capacity of the ecosystem is 1000 individuals. What is the instantaneous rate of change of the population?

- h) Prove that the steady state $x = K$ for the equation

$$\frac{dx}{dt} = rx \left\{ 1 - \left(\frac{x}{K} \right)^\theta \right\}, \text{ where } r, K, \text{ and } \theta \text{ are}$$

positive parameters, is stable.

- i) Consider the system $\frac{dr}{dt} = r(1-r)(r-2)(r-3)$,

$\frac{d\theta}{dt} = 1$, where (r, θ) be the polar coordinates on the plane. Find the number of limit cycles.

- j) Discuss Gompertz growth model.

- k) Explain the self-crowding effect in logistic growth model.

- l) What is diffusion in the context of mathematical modeling? How does diffusion differ from dispersal?

- m) State the basic assumptions of classic SIR compartmental model.

- n) What is meant by a hyperbolic fixed point?

2. Answer any **four** questions: $5 \times 4 = 20$

- a). The following model defined in $\Omega = \{(x, y) : x, y > 0\} \subset \mathbb{R}_+^2$ represents the densities of prey (x) and predator (y) populations at time t :

$$\frac{dx}{dt} = ax - bx^2 - \alpha xy,$$

$$\frac{dy}{dt} = y \left(e - \frac{sy}{x} \right),$$

where the parameters a, b, α, e , and s are all positive. Determine the existence and stability conditions of the coexistence equilibrium point.

- b) State and prove the Routh-Hurwitz criteria for a second order polynomial equation. Hence discuss the nature of the roots of characteristic equation of the differential

$$\text{equation } \frac{d^2x}{dt^2} + \frac{dx}{dt} + 3x = 0.$$

- c) Explain and analyze the standard Lotka-Volterra predator-prey model, including finding and discussing the stability of its isolated critical points.

- d) Reduce a two species diffusion model into a linearized system around any specially uniform steady state.

- e) Let the dimensionless activator-inhibitor system be described by

$$\frac{du}{dt} = a - u + u^2v,$$

$$\frac{dv}{dt} = b - u^2v,$$

where a, b are positive parameters. Identify the activator and inhibitor of the system. What biological behavior does the nonlinear system show? Are positive multi-stable states possible?

2+2+1

- f) Consider the following non-linear difference equation

$$x_{n+1} = \frac{\lambda x_n}{\mu + x_n},$$

where $\lambda > 0$ and $\mu > 0$. Find the fixed points and discuss their stability.

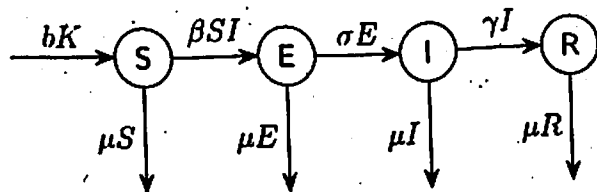
3. Answer any two questions: 10×2=20

- a) The population dynamics of a species is governed by the discrete model

$$N_{t+1} = N_t e^{r \left(1 - \frac{N_t}{K}\right)},$$

where r and K are positive parameters. Find the steady states and their stability. Show a period-doubling bifurcation at $r = 2$. 2+4+4

- b) From the flowchart of a SEIR model without vertical transmission



construct the SEIR model and find the basic reproduction number. Determine when the epidemic spreads or dies out. 3+3+4

- c) Write short notes on the following:

- | | |
|------------------------------|---|
| i) Michaelis-Menten kinetics | 5 |
| ii) Hopf bifurcation | 5 |