

U.G. 5th Semester Examination-2025**PHYSICS****[MAJOR-VII]****Course Code : PHY-M-T-7****(Quantum Mechanics)****[NEP-2020]**

Full Marks : 40

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.*

1. Answer any five questions: $2 \times 5 = 10$
- a) What do you mean by wave packet to describe a free particle with definite momentum?
 - b) A nucleus in its excited state returns to its ground state by emitting a gamma ray. If the lifetime of the excited state is $5.0 \times 10^{-12} \text{ s}$, calculate the uncertainty in the energy of the gamma ray.
 - c) Calculate the normalization constant A for a wave function given by (at $t = 0$)
$$\psi(x) = A \exp(ikx) \exp\left(-\frac{\sigma^2 x^2}{2}\right).$$
 - d) Evaluate the following commutation relations
$$[\hat{X}^2, \hat{P}_X] \text{ and } [\hat{P}_Y, \hat{L}_Z].$$
 - e) Write down the quantum mechanical Hamiltonian for one dimensional harmonic oscillator explaining all the terms.

[Turn over]

- f) What are $L - S$ and $J - J$ couplings of many electrons system?
- g) Determine the possible values of total angular momentum quantum number J in LS coupling of two atomic electrons having orbital quantum number $l_1 = 1, l_2 = 2$.
- h) What is the difference between Zeeman effect and Stark effect?

2. Answer any two questions: $5 \times 2 = 10$

a) What is conservation of probability density of a quantum system? Hence prove that the Hamiltonian operator in that case will be Hermitian. $2+3$

b) Define the expectation value of a dynamical variable in quantum mechanics. Find the expectation value of the kinetic energy for the $1s$ electron in the ground state of the hydrogen atom with the wave function

$$\psi(x) = \frac{1}{\sqrt{\pi a^3}} \exp\left(-\frac{r}{a}\right), \text{ where } a = \frac{\hbar^2}{me^2}. \quad 2+3$$

c) What is Anomalous Zeeman effect? Draw level diagrams showing anomalous Zeeman splitting for sodium D - line. $2+3$

d) Write down the zero point energy of quantum harmonic oscillator in one dimension. Hence show the uncertainty relation between Δx and Δp . $1+4$

3. Answer any two questions: $10 \times 2 = 20$

a) i) Explain what is meant by stationary state energy eigenfunctions. Prove that the energy eigenvalues of Hermitian operators are real. $1+2$

ii) Solve the time independent Schrodinger equation for free particle of mass m moving in the positive x - direction. Also calculate the position probability density for the particle. $2+1$

iii) A particle confined to a one-dimensional potential well has a wave-function given

$$\text{by } \psi(x) = \begin{cases} 0 & \text{For } x < -\frac{L}{2} \\ A \cos\left(\frac{3\pi x}{L}\right) & \text{For } -\frac{L}{2} < x < \frac{L}{2} \\ 0 & \text{For } x > \frac{L}{2} \end{cases}$$

Sketch the wave function?

Using Schrodinger equation $-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = E\psi$,

show that the energy $E = \frac{9\pi^2\hbar^2}{2mL^2}$. $2+2$

b) i) Write down the three cartesian components of angular momentum operator. Show the following commutation relations

$$[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z \text{ and } [\hat{L}^2, \hat{L}_z] = 0. \quad 2+3$$

ii) Define Lande' g factor? Calculate the total magnetic moment for Na atom in $L - S$ coupled state for the ground state $3^2S_{1/2}$ and the excited state $3^2P_{3/2}$? $1+4$

c) i) An electron of charge $-e$ and rest mass m is acted upon by the Coulomb attractive force of a point nucleus of charge $+e$.

Write down the Hamiltonian of the electron in spherical polar coordinates, assuming the nucleus is infinitely heavy then split up the Schrodinger equation into three separate equations for three variables r , θ and ϕ . 3+2

- ii) An electron is described by the wave function

$$\psi(x) = \begin{cases} 0 & \text{for } x < 0 \\ Ce^{-x}(1 - e^{-x}) & \text{for } x > 0, \end{cases}$$

where x is in nm and C is a constant. Determine the value of C that normalizes $\psi(x)$. 3

- iii) For what value of x is the probability of finding the electron the largest? 2

- d) i) Discuss briefly the Stern and Gerlach's experiment. What is the importance of this experiment? 3+1

- ii) How do you define the identical particles in quantum system? What are symmetric and antisymmetric wave functions? 1+2

- iii) Write down the Hamiltonian of a He atom as a two-electron system considering their mutual electrostatic interaction. 3