

## U.G. 2nd Semester Examination - 2020

## STATISTICS

## [HONOURS]

Course Code : STATH/CC-T-03

(Mathematical Analysis)

Full Marks : 60

Time :  $2\frac{1}{2}$  Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.*

1. Answer any **ten** questions:  $2 \times 10 = 20$
- State order axioms of the set of real numbers.
  - Define composite function of two real valued functions  $f$  and  $g$ .
  - Define upper bound of a set.
  - Define interior point of a set.
  - Define supremum of a subset in  $\mathbb{R}$ .
  - Find  $\lim_{n \rightarrow \infty} \sup$  and  $\lim_{n \rightarrow \infty} \inf$  of the sequence  $\left\{ \sin\left(\frac{n\pi}{2}\right) \right\}$ .
  - Does the series  $\sum_{n=1}^{\infty} \log\left(1 + \frac{1}{n}\right)$  converge or diverge? Justify.

- Define Cauchy sequence of real numbers with an example.
- Find the Taylor series expansion of  $\sin \pi x$  about  $x = \frac{1}{2}$ .
- Write  $n^{th}$  derivative of  $\sin(ax + b)$ .
- Discuss the applicability of Rolle's Theorem for the function  $f(x) = e^x \cos x$  in  $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ .
- Evaluate  $\lim_{x \rightarrow 0} \frac{x - \tan x}{x^3}$
- Evaluate  $\int \frac{dx}{(1-3x)\sqrt{x+2}}$
- State Monotone Convergence Theorem for sequences.
- Find the domain and range of the function  $f(x, y) = \sqrt{16 - 4x^2 - y^2}$

2. Answer any **four** questions:  $5 \times 4 = 20$
- Prove that every subset of a countable set is countable.
  - Prove that a sequence  $\{x_n\}$  defined on  $\mathbb{R}$  can converge to at most one point in  $\mathbb{R}$ .
  - Prove that the union of any collection of open sets is an open set.
  - If  $y = e^{ax} \cos^2 x \sin x$ , find  $y_n$ .

- e) Evaluate the integral by first changing the order of integration,  $\int_{x=0}^{x=3} \int_{y=x^2}^{y=9} x^3 e^{y^3} dy dx$ .
- f) Write down the usual polar transformation from  $(x, y)$  to  $(r, \theta)$ , where  $(x, y)$  lies in  $R \times R$ . Find the Jacobian of the transformation.

3. Answer any **two** questions:  $10 \times 2 = 20$

- a) i) Show that there exists a positive real number  $x$  such that  $x^2 = 2$ . Is  $x$  rational?
- ii) Show that  $\sup \left\{ 1 - \frac{1}{n} : n \in \mathbb{N} \right\} = 1$ .  $5+5$
- b) Prove that in Euclidean space  $R^k$ , every Cauchy sequence is convergent.  $10$
- c) i) State Lagrange's Mean Value Theorem and write its geometrical interpretation.
- ii) Using Lagrange's Mean Value Theorem show that

$$\left| \frac{\cos a\theta - \cos b\theta}{\theta} \right| \leq b - a, \text{ if } \theta \neq 0. \quad 5+5$$

- d) i) If  $y = (x + \sqrt{x^2 - a^2})^m$ . Prove that  $(x^2 - a^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$
- ii) Evaluate the integral  $\int_0^\infty x^6 e^{-2x} dx$ .  $5+5$

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