

U.G. 1st Semester Examination - 2025

PHYSICS

[MINOR]

Course Code : PHY-MI-T-01

(Mathematical Physics-I)

[NEP-2020]

Full Marks : 30

Time : 2 Hours

*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.*

GROUP-A

1. Answer any five questions: 1×5=5

- a) State Gauss's divergence theorem.
- b) Write down the order and degree of the differential equation $y = x \frac{dy}{dx} + 3 \frac{dx}{dy}$.
- c) Check the continuity of the function $f(x) = |x| - x$ at $x = 0$.
- d) Write down the Taylor series expansion of $f(x)$ around $x = a$.
- e) Find the projection of the vector $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$ on the vector $\vec{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$.

[Turn over]

- f) If $f(x, y, z) = 3x^2y - y^3z^2$, find $\vec{\nabla}f$ at the point $(1, -2, -1)$.
- g) Find the trace and eigenvalues of the matrix $\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$.
- h) Define Dirac delta function.

GROUP-B

2. Answer any three questions: $5 \times 3 = 15$
- a) Solve the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 2e^x$, subject to the boundary condition $x = 0, y = 1$ and $\frac{dy}{dx} = 1$. 5
- b) Verify Green's theorem in the plane for $\oint_C (xy + y^2) dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 5
- c) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{pmatrix}$. 5
- d) Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$. 5

- e) i) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2\hat{i} - \hat{j} - 2\hat{k}$.
- ii) If $\vec{A} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$, find $\text{curl } \vec{A}$. 3+2
- f) Prove the vector identity $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$. 5

GROUP-C

3. Answer any one question: $10 \times 1 = 10$
- a) i) Find the velocity and acceleration of a particle which moves along the curve $x = 2\sin 3t, y = 2\cos 3t, z = 8t$ at any time $t > 0$. Find also the magnitude of the velocity and acceleration.
- ii) Find the volume of the parallelepiped whose edges are given by $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{C} = 3\hat{i} - \hat{j} + 2\hat{k}$.
- iii) Represent delta function as a limit of a Gaussian function and rectangular function.
- iv) Prove $\delta(x^2 - a^2) = \frac{\delta(x-a) + \delta(x+a)}{2|a|}$. 3+2+2+3

- b) i) Evaluate $\iint_S \vec{F} \cdot \hat{n} \, dS$, where $\vec{F} = 4xz \hat{i} - y^2 \hat{j} + yz \hat{k}$ and S is the surface of the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$.
- ii) Find the particular integral of the differential equation $(D^2 + 1)y = \cos x$.
- iii) Find the primitive of the differential equation $xy - \frac{dy}{dx} = y^3 e^{-x^2}$. 3+3+4
-