

U.G. 5th Semester Examination - 2024**PHYSICS****[HONOURS]****Course Code : PHY-H-CC-T-11****(Quantum Mechanics & Applications)****[Old Syllabus]****Full Marks : 40****Time : 2½ Hours***The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.***GROUP-A**

1. Answer any five questions: 2×5=10
 - a) The 2p state for the hydrogen atom is known to be $re^{\frac{-r}{2a_0}}(\cos\theta)$. Find out the expectation value of r in this state.
 - b) The operator $(x + \frac{d}{dx})$ has the Eigen value ∞ . Derive the corresponding Eigen function if $\Psi = \Psi_0$ at $x = 0$.
 - c) Prove that the energy eigenfunction of a free particle is doubly degenerate.
 - d) Calculate the commutator $[x^2, P_x^2]$.
 - e) What is Paschen Back effect?

[Turn over]

- f) Prove that $\exp [i(AB - BA)]$ is a Hermitian operator if A, B are Hermitian operators.
- g) Find out the magnetic moment of an atom in the state $^2D_{3/2}$.
- h) Which of these elements H, He, Li, Mg, Na exhibit only normal Zeeman effect and why?

GROUP-B

2. Answer any two questions: $5 \times 2 = 10$
- a) How many spectral lines appear in the Zeeman splitting of the $^2D_{3/2} - ^2P_{1/2}$ transition of Sodium, Explain. 5
- b) For the normal Zeeman effect of hydrogen, explain how the Lorentz triplet occurs. How are the π and σ lines polarized? 2+3
- c) For an operator $\hat{A}$ corresponding to an observable A prove that

$$\frac{d}{dt} \langle A \rangle = \langle \frac{\partial \hat{A}}{\partial t} \rangle + \frac{\langle [\hat{A}, H] \rangle}{i\hbar} \quad 5$$

- d) Consider the finite square well potential

$$V(x) = \begin{cases} -V_0 & \text{for } -a < x < a \\ 0 & \text{for } |x| > a \end{cases}$$

where V_0 is a positive constant and $2a$ is the width of the potential well. Derive the transcendental equation determining the discrete energy eigenvalues for symmetric wave functions (bound states). 5

GROUP-C

3. Answer any two questions: $10 \times 2 = 20$

- a) Calculate the Lande's g factor for 3S_1 and 3P_1 levels. Hence estimate the energy splitting of the two levels if a magnetic field of 1 T is applied. How many spectral lines will arise from the anomalous Zeeman splitting due to the transition between these levels? Draw a neat diagram showing these transitions.

$$3+2+3+2$$

- b) i) 'A free particle does not have definite energy'— Explain.
- ii) Consider a linear harmonic oscillator for which the total energy is given by

$$E = \frac{p_x^2}{2m} + \frac{1}{2} m \omega^2 x^2$$

where the symbols have their usual meanings. The particle is assumed to be confined to a region $\sim a$. Use the uncertainty principle to obtain the minimum (ground state) of the oscillator.

iii) An electron of energy 342 eV is confined in a one-dimensional box of length 1 Å. Find out the quantum number of the energy level of the electron and the energy needed to excite it in the next higher level.

iv) If the parity operator $\hat{p}$ satisfies $\hat{p}\psi(x) = \psi(-x)$, show that $\hat{p}$ has only two eigenvalues 1 and -1. Find the expectation value for each of them.

2+3+3+2

c) A particle in an infinite square well a as its initial Wave function an equal mixture of the first two stationary states:

$$\psi(x, 0) = A[\psi_1(x) + \psi_2(x)].$$

Assume $\psi_n(x) = \sqrt{2/a} \sin \frac{n\pi x}{a}$ and $E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}$.

i) Normalize $\psi(x, 0)$.

ii) Find $\psi(x, t)$ and $|\psi(x, t)|^2$.

iii) Compute $\langle x \rangle$. What is the angular frequency and amplitude of the oscillation?

iv) Find $\langle H \rangle$. $2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2}$

d) i) Consider a particle of mass m moving in a one-dimensional potential specified by

$$V(x) = \begin{cases} 0, & -2a < x < 2a \\ \infty, & \text{otherwise} \end{cases}$$

Find the energy eigenvalues and eigenfunctions.

ii) Prove the following commutation relations

$$[x, p_x] = [y, p_y] = [z, p_z] = i\hbar$$

$$[p_x, p_y] = [p_y, p_z] = [p_z, p_x] = 0$$

iii) Write the time-dependent Schrodinger equation for a free particle in the momentum space and obtain the form of the wave function.

4+3+3