

R-8

U.G. 4th Semester Examination - 2024

PHYSICS**[HONOURS]****Course Code : PHY-H-CC-T-8****(Mathematical Physics-3)**

Full Marks : 40

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.***GROUP-A**

1. Answer any five questions:

 $2 \times 5 = 10$

- a) Show that $z = i^{-2i}$ is a real number and determine its value.
- b) Determine the type of singularities (if any) possessed by the functions $f(z) = \sinh(\frac{1}{z})$ at $z = 0$ and $z = \infty$.
- c) Expand the function $f(z) = e^{-iz}$ in Taylor series.
- d) Check whether the function $f(z) = az^2$ (where a is any constant) is analytic everywhere or not?
- e) Determine Fourier transform of $f(x) = e^{-|x|}$.

[Turn over]

- f) What is inverse Fourier transform?
- g) If the Laplace transform of a function $f(t)$ is given by $f(s)$ then determine the Laplace transform of the function $f(ct)$ where c is a constant.
- h) Show that the Laplace transform of the function $f(t) = t^2 \sin(t)$ is given by $\frac{2(3s^2-1)}{(s^2+1)^3}$.

GROUP-B

2. Answer any two questions: 5×2=10

- a) Prove that if a complex function $f(z)$ is analytic in a domain R then the function must be independent of $\bar{z}$. Evaluate the following integral using Cauchy's integral formula over a unit circle in counter clockwise direction.

$$\oint \frac{dz}{z(z+2)} \quad 2+3$$

- b) Find the branch points of $f(z) = \sqrt{z^2+1}$ and draw the branch cut. 5
- c) Express Dirac delta function in terms of its Fourier transform. Prove that $\delta(ax) = \frac{\delta(x)}{|a|}$. 2+3
- d) State and prove the convolution theorem for Laplace transforms. 1+4

GROUP-C

Answer any two questions:

10×2=20

3. a) Evaluate the following integral over the closed contour given by $|z| = \frac{1}{2}$. 4

$$\oint_C \frac{(2z+1)}{z^2+z} dz$$

- b) Find the Laurent series of $f(z) = \frac{1}{z(z-2)^3}$ about $z = 0$. Hence verify that $z = 0$ is a simple pole (order 1). Find the residue of $f(z)$ at the pole. 4+1+1

4. a) State and prove residue theorem.
- b) Solve the following integral using residue theorem.

$$\int_0^{2\pi} \frac{d\theta}{1+a \cos \theta} \quad |a| < 1 \quad 5+5$$

5. a) Find the Fourier transform of the following function

$$f(x) = \frac{1}{a\sqrt{2\pi}} e^{-\frac{t^2}{2a^2}} \quad -\infty < t < \infty$$

- b) Find the convolution $f(x) = \cos(x)$ and $g(x) = e^{-a|x|}$ where $a > 0$. 5+5
6. a) Find the Laplace transform of the following function

$$f(t) = \begin{cases} e^t & 0 < t < 1 \\ 1 & t > 1 \end{cases}$$

- b) Solve the following initial value problem using Laplace transform

$$y'' - 10y' + 9y = 5t, \quad y(0) = -1, y'(0) = 2$$

5+5