

**U.G. 5th Semester Examination - 2024**

# **MATHEMATICS**

**[HONOURS]**

**Course Code : MATH-H-CC-T-12**

**(Mechanics-I)**

**[New Syllabus]**

**Full Marks : 60**

**Time :  $2\frac{1}{2}$  Hours**

*The figures in the right-hand margin indicate marks.*

*Symbols and notations have their usual meanings.*

1. Answer any ten questions: 2×10=20

- a) A point moves in a straight line so that its distance  $x$  from a fixed point at time  $t$  is proportional to  $t^n$ . If  $v$  be the velocity and  $f$  the acceleration at any time  $t$ , show that  $(n-1)v^2 = nfx$ .
- b) A particle moves along a straight line according to the law  $x^2 = at^2 + 2bt + c$ . Prove that the acceleration varies as  $\frac{1}{x^3}$ .

*[Turn over]*

- c) If the position of a moving point is given by  $x = a \cos t$  and  $y = b \sin 2t$ , find the path.
- d) If the radial and transverse velocities of a particle are always proportional to each other, show that the path is an equiangular spiral.
- e) A particle describes a circle under a force to a fixed point on the circumference. Find the law of force.
- f) A bullet of mass  $m$  moving with a velocity  $u$ , strikes a block of mass  $M$ , which is free to move in the direction of motion of the bullet and is embedded in it. Show that the loss of K. E. is  $\frac{1}{2} \frac{mM}{M+m} u^2$ .
- g) An engine of power  $P$  draws up a load of mass  $m$  hanging vertically at the end of a light inextensible rope. Show that if  $v$  is the speed of the load, then  $\frac{dv}{dt} = \frac{P}{mv} - g$ .
- h) A particle moving vertically upwards under gravity is met with air resistance equal to  $k$  times its velocity at any time. Write down the differential equation for its height  $x$  at any time  $t$ .

- i) The radius of the earth is  $\lambda^2 g$  where  $g$  is the acceleration due to gravity at the earth's surface. Show that the velocity of a satellite of the earth moving in a circular orbit whose radius is approximately equal to that of the earth is  $\lambda g$ .
- j) The speed  $v$  of a point moving along the  $x$ -axis is given by  $v^2 = 16 - x^2$ . Prove that the motion is simple harmonic motion and find its amplitude.
- k) How does a rigid body differ from a deformable body?
- l) Obtain the relation between the rate of change of angular momentum of a moving particle and force acting on it.
- m) State D'Alembert's principle.
- n) Find the moment of inertia of the perimeter of a circle about a tangent.
- o) Explain the terms "apse" and "apsidal angle" related to a central orbit.

2. Answer any four questions:

$5 \times 4 = 20$

- a) An engine working at a constant rate  $H$  draws a load of mass  $M$  against a resistance  $R$ . Show that the maximum speed attained is  $H/R$  and the time taken to attain half this speed is  $\frac{MH}{R^2} \left[ \log 2 - \frac{1}{2} \right]$ .

- b) A shot of mass  $m$  penetrates of thickness  $k$  of a fixed plate of mass  $M$ . Prove that if the plate be free to move, the thickness penetrated is

$$\frac{k}{\left(1 + \frac{m}{M}\right)}.$$

- c) A particle moves from rest at a distance from a fixed point O under the repulsive force  $\frac{\mu}{(\text{distance})^2}$  per unit mass. Determine its velocity and time.
- d) A particle moves in a plane with an acceleration which is always towards and perpendicular to a fixed straight line in the plane and varies inversely as the cube of the distance from it. If the particle be projected with a given velocity from a point distant  $a$  from the fixed straight line, determine its path.
- e) Four forces each of magnitude  $F$  act on a rigid body, three of forces act along the rectangular Cartesian co-ordinate axes  $x$ ,  $y$  and  $z$ , while the fourth force acts along the line  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ . Find the equation of the central axis.

- f) Define degrees of freedom. Show that the rate of change of angular momentum of a rigid body about the axis of rotation is equal to the sum of moments about the same axis of all forces acting on the body.

3. Answer any two questions:

$$10 \times 2 = 20$$

- a) i) What do you mean by principal axes? Find the condition that a given straight line may be a principal axis at any point on the line and if the line is a principal axis then determine the other two principal axes.
- ii) Show that the momental ellipsoid at the centre of an elliptic plate is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + z^2 \left[ \frac{1}{a^2} + \frac{1}{b^2} \right] = \text{constant}.$$

$$(1+5)+4=10$$

- b) i) If the law of force be  $2\mu(u^3 - a^2u^5)$  and the particle be projected from an apse at a distance  $a$  with velocity  $\frac{\sqrt{\mu}}{a}$ , show that it will be at a distance  $r$  from the centre after a time  $\frac{1}{2\sqrt{\mu}} \left[ r\sqrt{r^2 - a^2} + a^2 \cosh^{-1} \left( \frac{r}{a} \right) \right]$ .

- ii) A particle moves in one plane under a fixed straight line on the path. Its magnitude being  $\mu \div (\text{distance from the line})^2$ . If initially it be at a distance  $2a$ , from the line and be projected with a velocity  $\frac{\sqrt{\mu}}{a}$  parallel to the line, prove that the path traced out is a cycloid.  $5+5=10$

- c) i) Suppose a particle is moving in a plane. Besides the central acceleration  $F$  it has an acceleration  $T$  in the direction perpendicular to  $F$ . Show that the differential equation of the path is

$$\frac{d^2u}{d\theta^2} + u = \frac{F - \frac{T}{u} \frac{du}{d\theta}}{h^2 u^2}.$$

- ii) If  $a$  be the amplitude and  $T$  be the period of a particle executing SHM in a straight line, show that the time taken by the particle to travel a distance  $x$  from the

$$\text{centre of force is } \frac{T}{2\pi} \sin^{-1} \left( \frac{x}{a} \right).$$

$6+4=10$

- d) i) A particle is projected from a point  $p$  with velocity  $v$  in a direction making an angle  $\alpha$  with the radius vector. If  $v$  were  $v'$ , the orbit would be a parabola. Prove that if the orbit is hyperbolic the angle between the asymptotes is

$$2 \tan^{-1} \left[ \frac{2v}{v'} \left( \frac{v^2}{v'^2} - 1 \right)^{\frac{1}{2}} \sin \alpha \right].$$

- ii) A particle is projected vertically upwards with velocity  $V$ , and the resistance of the air produces a retardation  $kv^2$ , where  $v$  is the velocity. Show that the velocity  $V_1$  with which the particle will return to the point of projection is given by  $\frac{1}{V_1^2} = \frac{1}{V^2} + \frac{1}{W^2}$  where  $W$  is the terminal velocity.

$5+5=10$