

U.G. 1st Semester Examination - 2024

MATHEMATICS

[MINOR]

Course Code : MATH-MI-T-1

(Algebra and Analytical Geometry)

[NEP-2020]

Full Marks : 40

Time : 2 Hours

*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**The notations and symbols have their usual meanings.*1. Answer any five questions: $2 \times 5 = 10$

a) Give the polar representation of the complex

number : $z = -\frac{1}{4} + i\frac{\sqrt{3}}{4}$.

b) Does the equation $x^2 - y^2 = 0$ represent a pair of straight lines? If so, find the angle between them.c) Compute AB where $A = \begin{pmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{pmatrix}$ and

$$B = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}.$$

[Turn over]

d) How many points are there on the ellipse $\frac{z}{r} = 1 - \frac{1}{2} \cos \theta$ whose radius vector is 4? Find the points.

e) Find the centre of the conic $x^2 - 2xy + 8y^2 + x - y + 5 = 0$.

f) Define a relation ρ on Z by $a\rho b$ if and only if $3a + 4b = 7n$, for some integer n . Show that ρ is reflexive and symmetric.

g) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 0 & 1 & -1 \end{pmatrix}$.

h) Find the value of $\alpha^2 + \beta^2 + \gamma^2$ if α, β, γ are the roots of $x^3 - 6x^2 + 11x - 6 = 0$.

2. Answer any two questions: $5 \times 2 = 10$

a) Find a solution (if exists) of the following system of linear equations:

$$x - 3y + 4z = -4$$

$$3x - 7y + 7z = -8$$

$$-4x + 6y - z = 7.$$

b) Show that $n^n > 1.3.5 \dots (2n-1)$.

c) Prove that the equation $ax^2 + 2hxy + by^2 = 0$, $(a, h, b) \neq (0, 0, 0)$ will be a pair of straight lines through the origin if and only if $h^2 - ab \geq 0$.

d) A sphere touches the planes $2x + 3y - 6z + 14 = 0$ and $2x + 3y - 6z + 42 = 0$ and its centre lies on the line $2x + z = 0, y = 0$. Find the equation of the sphere.

3. Answer any two questions: $10 \times 2 = 20$

a) i) If $\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma = 0$ then prove that $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$. 5

ii) If α, β, γ are the roots of the equation $x^3 - 7x^2 + x - 5 = 0$, find the equation whose roots are $\alpha + \beta, \beta + \gamma, \alpha + \gamma$. 5

b) i) Let a and b be nonzero real numbers.

Define $f: R \rightarrow R$ by $f(x) = \frac{ax}{x+b}$. Prove

that f is one-to-one. Are there solutions to the equations $f(x) = a$ and $f(x) = b$? $2+2$

ii) What do you mean by a subgroup of a group? Do you find a group structure in Z ? If so then check whether N and $H = \{2n: n \in Z\}$ are subgroups of Z or not. Here Z = set of all integers and N = set of natural numbers. $1+1+2+2$

c) i) If the expression $ax^2 + 2hxy + by^2$ is transformed to $a_1x_1^2 + 2h_1x_1y_1 + b_1y_1^2$ by transformation of rotation then prove the following $ab - h^2 = a_1b_1 - h_1^2$. 4

ii) Show by reducing to the canonical form, the equation $x^2 - 2xy + y^2 + 6x - 14y + 29 = 0$ actually represents a parabola. 6

d) i) Find the value of m , so that the lines $\frac{x-1}{2} = \frac{y-4}{1} = \frac{z-5}{2}$ and $\frac{x-2}{-1} = \frac{y-8}{m} = \frac{z-11}{4}$ may intersect. 5

ii) Find the image of x -axis on the plane $x + 2y + 3z = 14$. 5
