

U.G. 3rd Semester Examination - 2024**PHYSICS****[HONOURS]****Course Code : PHY-H-CC-T-05****(Mathematical Physics-II)****[CBCS]**

Full Marks : 40

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.*

1. Answer any five questions:

 $2 \times 5 = 10$

- a) Form the partial differential equation from $z = \phi(x + cy) + \psi(x - cy)$.
- b) What are the importance of Fourier's theorem?
- c) Find the expression for the maximum permissible error in y for the relation $y = a/b$.
- d) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.
- e) Define error function.

[Turn over]

- f) What is regular singular point of a differential equation?
- g) What do you mean by systematic error?
- h) State Rodrigue's formula for Legendre polynomial.

2. Answer any two questions: 5×2=10

- a) Show that

$$J_1(x) - J_3(x) + J_5(x) - J_7(x) + \dots = \frac{1}{2} \sin x. \quad 5$$

- b) An alternating current passing through a rectifier has the form 5

$$I(\theta) = \begin{cases} I_0 \sin \theta, & \text{for } 0 < \theta < \pi \\ 0, & \text{for } \pi < \theta < 2\pi \end{cases}$$

Find the Fourier series of the function $I(\theta)$.

- c) Prove that for Bessel function

$$J_m(x) = \frac{1}{\pi} \int_0^\pi \cos(m\theta - x \sin \theta) d\theta. \quad 5$$

- d) i) Show that $B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

ii) Evaluate $\int_0^1 \frac{x^3 dx}{\sqrt{1-x}}$. 3+2

3. Answer any two questions: 10×2=20

- a) i) Obtain the Fourier series of $f(x)$, where

$$f(x) = \begin{cases} \cos x & \text{for } 0 \leq x \leq \pi \\ -\cos x & \text{for } -\pi \leq x \leq 0 \end{cases}$$

- ii) If $y = x^2$ over the range $x = -l$ to $x = l$, find a Fourier series expansion for y for points within the above mentioned limits.

- iii) Show that

$$\int_{-1}^1 P_m(x) P_n(x) dx = 0 \text{ when } m \neq n. \quad 4+4+2$$

- b) A rectangular membrane of sides a and b , having edges parallel to x and y axes and bounded rigidly at the edges is given by a slight deformation in the z -direction perpendicular to its plane. The differential equation for z is $\frac{\partial^2 z}{\partial t^2} = c^2 \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right)$, where c is a constant. Solve the equation by the method of separation of variables, assuming the initial conditions at $t = 0$, $z = f(x, y)$ and $\frac{dz}{dt} = 0$. Find the frequencies of the natural vibrations of the membrane. What is the fundamental frequency? 8+1+1

- c) Evaluate

i) $\int x^3 J_2(x) dx$

ii) $\int x P_n(x) P_{n-1}(x) dx$



iii) $\int_{-1}^{+1} P_n^2(x) dx$ using Rodrigue's formula.

3+3+4

d) i) Prove the following recurrence relation:

$$nP_n(x) = xP'_n(x) - P'_{n-1}(x).$$

ii) Prove $J_{n-1}(x) - J_{n+1}(x) = 2J'_n(x)$.

iii) Prove that $\int_{-\infty}^{+\infty} e^{-x^2}(x) dx = 2^n \sqrt{\pi} \cdot n!$.

3+3+4