

U.G. 4th Semester Examination - 2022

STATISTICS

[HONOURS]

Generic Elective Course (GE)

Course Code : STAT-H-GE-T-4

(Introductory Probability)

Full Marks : 50(40+10)

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.***Answer all the questions.**1. Answer any **five** questions: $2 \times 5 = 10$

- State the axiomatic definition of probability.
- If a Poisson distribution with parameter m has mode at $X = 3$ and $X = 4$ then what is the value of m ?
- In a Binomial distribution with mean = 6, variance = 4, find the values of the parameters.
- If A and B are independent events, show that A^c and B^c are also independent.

- If X follows $N(\mu, \sigma^2)$ then what is the expression for $P(a < X < b)$ in terms of the cumulative distribution function of standard normal variate?
- If X is a continuous random variable then what is the value of $P[X=c]$?
- for a Poisson distribution with parameter 2, find the value of $P(X > 1)$.
- State the theorem of total probability.

2. Answer any **two** questions: $5 \times 2 = 10$

- If A and B are not mutual exclusive, then show that $P(AB) \geq P(A) + P(B) - 1$. The probability that Asoke can solve a problem is $4/5$, Amal can solve the same is $2/3$ and Rabi can solve it is $3/7$. All of them try independently, what is the probability that the problem is solved?
- A continuous random variable has the following probability density function

$$f(x) = \begin{cases} 0 & \text{for } x \leq 2 \\ k(x-2)^3 & \text{for } 2 \leq x \leq 4 \\ 1/3 & \text{for } 4 \leq x \leq 5 \\ 0 & \text{for } x > 5 \end{cases}$$

Obtain the distribution function $F(x)$ and also find $P[2 < 3 < 4]$.

- c) Find the mode of a Poisson distribution with parameter λ .
- d) The random variables $X_1, X_2, \dots$ are independent and X_i assumes just two values $-1/2^i$ and $1/2^i$ with equal probabilities. Show that the weak law of large numbers holds for these variables.

3. Answer any **two** questions: 10×2=20

- a) i) Define conditional probability. State and prove the theorem of compound probability. 6
- ii) A man draws 2 balls from a bag containing 3 white and 5 red balls. If he is to receive Rs. 14 for every white ball which he draws and Rs. 7 for every red ball. What is his expectation? 4
- b) i) Find a recursion relation among central moments for a Binomial random variable. 5
- ii) State and prove weak law of large numbers. 5

- c) When is a random variable said to follow hypergeometric distribution with parameters N , p and m ? Give one example where this distribution is appropriate. Derive the mean of this distribution also show that this distribution tend to the binomial distribution when $N \rightarrow \infty$.
- d) i) Derive the moment generating function about mean of a normal distribution with parameters μ and σ^2 . Hence show that all odd order central moments of this distribution are zero. 6
- ii) Write a note on Central Limit Theorem. 4

[Internal Assessment: 10]
