

U.G. 6th Semester Examination - 2025

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-03A

(Fuzzy Set Theory)

[New Syllabus]

Full Marks : 60

Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

The symbols and notations have their usual meanings.

1. Answer any ten questions:

2×10=20

- i) Find $A \cdot B^{-1}$, where the interval numbers are given by $A = [-7, 4]$ and $B = [2, 12]$.
- ii) Find the distance between two interval numbers, $A = [-6, 8]$ and $B = [3, 10]$.
- iii) Define two level interval numbers.
- iv) For any $A \in \mathcal{F}(X)$ and $\alpha, \beta \in [0, 1]$, find the relationship between $\alpha^+ A$ and $\beta^+ A$ for $\alpha \leq \beta$.
- v) Give an example of a fuzzy similarity relation with justifications.
- vi) Find the standard fuzzy complement of

$$A(x) = \frac{x}{x+3} \text{ defined on } \{0, 1, 2, 3, 4, 5\}.$$

[Turn Over]

vii) Show by an example that standard intersection of two normal fuzzy sets may not be normal.

viii) Using the concept of symmetric triangular fuzzy number, describe 'about - 4'.

ix) Using standard fuzzy union and standard fuzzy intersection, find $A \cup B$ and $A \cap B$, where

$$A = \frac{0.8}{0} + \frac{0}{3} + \frac{0.6}{4} + \frac{0.2}{6} \text{ and } B = \frac{0.4}{0} + \frac{1}{3} + \frac{0.5}{4} + \frac{0.2}{6}$$

x) State 2nd decomposition theorem.

xi) Is every fuzzy set a fuzzy number? Justify.

xii) Find all α -cuts of the fuzzy set

$$A = \frac{0.5}{x_1} + \frac{0.9}{x_2} + \frac{0.2}{x_3}$$

xiii) Find the relationship between max-min composition and relational join.

xiv) Give an example of a similarity relation.

xv) Is every fuzzy number convex? Justify.

2. Answer any four questions: $5 \times 4 = 20$

i) Check whether $A(B + C) = AB + AC$ for the interval numbers, $A = [-1, 3]$, $B = [2, 5]$ and $C = [-2, 7]$.

ii) Using extension principle define $f(A)$, where A is a fuzzy set and f is a function. Find the relationship between ${}^\alpha f(A)$ and $f({}^\alpha A)$ Justify whether those are equal.

iii) Find strong $(\alpha+)$ -cut, height and support of the following fuzzy set

$$A(x) = \begin{cases} \frac{x-6}{15} & \text{if } 6 \leq x \leq 21 \\ \frac{35-x}{14} & \text{if } 21 \leq x \leq 35 \\ 0 & \text{Otherwise} \end{cases}$$

iv) Find the standard max-min composition $P \circ Q$ of the following two binary fuzzy relations P and Q :

$$P = \begin{matrix} & a & b & c \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0.1 & 0.7 & 0 \\ 0.9 & 0 & 0 \\ 0 & 0.8 & 0.4 \\ 0.2 & 0 & 0.4 \end{bmatrix} \end{matrix} \text{ and } Q = \begin{matrix} & \alpha & \beta \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 0.6 & 0.7 \\ 0.9 & 0.4 \\ 0.3 & 0.9 \end{bmatrix} \end{matrix}$$

v) Let A and B be fuzzy sets defined on the universal set $\mathbb{Z}$ whose membership functions are given by $A(x) = 0.8/(-2) + 0.6/0 + 0.3/1 + 0.7/3$ and $B(x) = 0.4/2 + 1/3 + 0.9/6 + 0.1/8$. Also, let a function $f: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ be defined for all $x_1, x_2 \in \mathbb{Z}$ by $f(x_1, x_2) = x_1 + x_2$. Calculate $f(A, B)$.

vi) Find the transitive max-min closure of the fuzzy

$$\text{relation: } R = \begin{bmatrix} 0.7 & 0.4 & 0 & 0.3 \\ 0 & 0 & 0.6 & 1 \\ 0 & 0.2 & 0 & 0.3 \\ 0 & 0.2 & 0.7 & 0 \end{bmatrix}$$

3. Answer any two questions:

10×2=20

- i) a) State and prove second decomposition theorem.
b) Solve the fuzzy equation: $AX=B$ where A, B are given by

$$A(x) = \begin{cases} x-3 & \text{if } 3 \leq x \leq 4 \\ 5-x & \text{if } 4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases} \text{ and } B(x) = \begin{cases} \frac{x-12}{8} & \text{if } 12 \leq x \leq 20 \\ \frac{32-x}{12} & \text{if } 20 \leq x \leq 32 \\ 0 & \text{otherwise.} \end{cases}$$

(2+3)+5

- ii) a) For the interval numbers, $A = [2,10]$, $B = [8,16]$ and $C = [1,18]$, verify the triangle inequality $d(A, C) + d(B, C) \geq d(A, B)$ where d stands for distance.

- b) Show by an example that if a fuzzy binary relation R is reflexive, then $R \subset R \circ R$.

5+5

- iii) a) Establish the relationship between $\bigcup_{i \in I} {}^{\alpha} A_i$ and ${}^{\alpha}(\bigcup_{i \in I} A_i)$, where $A_i \in \mathcal{F}(X)$ for all $i \in I$, where I is an index set.

- b) Let A and B be fuzzy sets defined on the universal set $\mathbb{Z}$ whose membership functions are given by $A(x) = 0.6/(-2) + 1/0 + 0.7/1 + 0.4/3$ and $B(x) = 0.4/2 + 1/3 + 0.3/6 + 0.1/8$. Also, let a function $f: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ be defined for all $x_1, x_2 \in \mathbb{Z}$ by $f(x_1, x_2) = x_1 + x_2$. Calculate $f(A, B)$.

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