

U.G. 1st Semester Examination - 2021

STATISTICS

[HONOURS]

Course Code : STAT-H-CC-T-2

(Probability and Probability Distributions-I)

Full Marks : 50 (40+10)

Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer any **five** questions: 2×5=10
 - a) In a group of 5 boys, what is the probability that there will be at least one matching birthday among them?
 - b) Write down the classical definition of probability.
 - c) Define pairwise independence and mutual independence.
 - d) Write down the multiplication rule of probability for four events.
 - e) Define cumulative distribution function. Write down its properties.

- f) State Poincare's theorem.
- g) If X and Y are independent show that the variance of their sum is equal to the sum of their variances. Is the converse true?
- h) Let $A_1, A_2, \dots, A_k$ be a set of k independent events with $P(A_i) = 1 - q_i$, $i = 1(1) k$. Show that $P(\bigcup A_i) = 1 - q_1 \cdot q_2 \dots q_k$.

2. Answer any **two** questions: 5×2=10
 - a) When is a random variable X said to follow Poisson distribution? If X follows Poisson distribution with parameter λ , find $E\{1/(1+X)\}$.
 - b) Define conditional probability. Let X_n be the number of heads obtained in first n trials of tossing a perfect coin. Find $P(X_n = x | X_{n-1} = y)$ for $x \geq y$.
 - c) Let X be a non-negative random variable with cdf F. Show that

$$E(X) = \int_0^{\infty} [1 - F(x)] dx.$$
 - d) Write down the p.m.f. of a Hypergeometric distribution and derive its mean.

3. Answer any **two** questions: $10 \times 2 = 20$

- a) A number denoted by X is chosen at random from the set of integers $\{1, 2, 3, 4\}$. Another number Y is chosen at random from the set $\{1, 2, \dots, X\}$.

(i) For each integer i from 1 to 4, Find $P(Y=1|X=i)$, (ii) Find $P(Y=1)$, (iii) Find $P(X=2|Y=1)$. $3+3+4$

- b) Let the probability that a family has exactly n children be αp^n ($\alpha > 0, 0 < p < 1$). Suppose all sex distributions of n children have the same probability.

i) Show that for $k \geq 1$, the probability that a family has exactly k boys is $2\alpha p^k / (2-p)^{k+1}$.

ii) Given that a family has at least one boy, what is the probability that there are two or more boys? $5+5$

- c) i) State Bayes' theorem and mention one of its applications.

ii) Let A and B be two events such that $P(A)=p_1 > 0$, $P(B)=p_2 > 0$ and $p_1 + p_2 > 1$. Show that $P(B|A) = 1 - [(1-p_2)/p_1]$. $5+5$

- d) i) Give an example of a joint distribution of X and Y such that $\text{Cov}(X, Y) = 0$, but X and Y are not independent.

ii) Write down the p.m.f. of a non-singular trinomial distribution and derive the conditional distribution of any component given the others. $4+6$

[Internal Assessment : 10]
