

U.G. 6th Semester Examination-2025**STATISTICS****[PROGRAMME]****Discipline Specific Elective (DSE)****Course Code : STAT-G-DSE-T-1B&1D****Full Marks : 50(40+10)****Time : 2½ Hours***The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meanings.***Answer all the questions from Selected Option.****OPTION-A****STAT-G-DSE-T-1B****(Survival Analysis)**

1. Answer any five questions: 2×5=10
 - a) Define survival function and hazard function.
 - b) Given that the hazard function takes the constant value unity, derive its survival function and probability density function.
 - c) What is meant by censored data? Explain with an example.

[Turn over]

- d) What do you mean by NBUE?
- e) What is delta method?
- f) Consider the survival function of Weibull distribution,

$$S(t) = e^{-(\lambda t)^\alpha}, t \geq 0; \alpha, \lambda > 0.$$

Find the hazard function.

- g) What is simple epidemic model?
- h) What do you mean by a random censoring scheme?

2. Answer any two questions: 5×2=10

- a) Describe the different parts of a bathtub curve.
- b) Find the relationship between survival function and MRL.
- c) Describe Type-I censoring.
- d) Describe modified minimum chi-square method of estimation.

3. Answer any two questions: 10×2=20

- a) i) Find the maximum likelihood estimators of the parameters of a Weibull distribution under Type-II censoring scheme.
- ii) Under random censoring, find the maximum likelihood estimator of the parameter of an exponential distribution. 5+5
- b) Describe competing risk. 10

- c) i) Define DFR, DFRA, NWU, NWUE and DMRL.

- ii) Show that $DFR \Rightarrow DFRA \Rightarrow NWU \Rightarrow NWUE$.

5+5

- d) Using life table method, find an estimator of the survival function. And also find its variance.

5+5

[Internal Assessment: 10]

OPTION-B
STAT-G-DSE-T-1D
(Actuarial Statistics)

1. Answer any five questions: 2×5=10

- a) Explain the term 'effective rate of discount'.
- b) Explain the concept: Speculative risk.
- c) State any two characteristics of insurable risk.
- d) If an amount of Rs. 1,000,000 is invested on January 1, 2024, at a constant force of interest of 0.06 per annum, determine the accumulated value as of January 1, 2025.
- e) What are the key functions performed by actuaries in the insurance industry?
- f) A sum of Rs. 5,000 is invested in a savings plan that matures to Rs. 7,500 in 10 years. Find the rate of interest per annum assuming it is compounded annually.
- g) Find the present value of annuity certain immediate at the rate of 1 unit per annum for n years.
- h) Explain: n year term insurance.

2. Answer any two questions: 5×2=10

- a) Explain (i) Utility function, (ii) Expected utility criterion.
- b) Mortality rates (q_x) for a certain population of insects are as follows:

Age in years (x)	0	1	2	3	4	5	6
q_x	0.1	0.2	0.3	0.5	0.6	0.8	1

Construct the columns of life table l_x , L_x , T_x for a radix of 10000.

- c) Let X have a discrete distribution as follows:

$$P(X = 100) = 0.5$$

$$P(X = 175) = 0.3$$

$$P(X = 200) = 0.2$$

If the insurer effects excess of loss reinsurance with retention level 150, what is the distribution of the non-zero payments made by the reinsurer?

- d) A loan of Rs. 2500000 is taken and it has to be repaid in 20 equal installments payable yearly at the beginning of the year. Based on 8% annual effective rate of interest determine the amount of installments.

3. Answer any two questions: 10×2=20

- a) i) If for the annuity certain the payments are made regularly at the beginning of the year then derive that

$$\ddot{S}_{n\overline{i}} = (1+i)^n \ddot{a}_{n\overline{i}}$$

- ii) Examine whether the following functions can serve as survival functions for $x \geq 0$.

A. $S(x) = \exp(x - 0.7(2^x - 1))$

B. $S(x) = \frac{1}{(1+x)^2}$

C. $S(x) = \exp(-x^2)$

4+6

- b) i) Suppose the life length random variable is modeled by a distribution with force of mortality as specified below:

$$\mu_s = \begin{cases} 0.01 & \text{if } 0 < s < 15 \\ 0.02 & \text{if } 15 \leq s \leq 25 \\ 0.03 & \text{if } s \geq 25 \end{cases}$$

Find corresponding $S(x)$, $f(x)$ and probability density function $g(t)$ of $T(20)$.

- ii) If $\mu_x = 0.02$ for $40 \leq x \leq 47$ evaluate $2^{p_{40}}$ and $3/4^{q_{40}}$. 6+4

- c) i) Explain the term 'Annuity' with an illustration.
- ii) An insurer is considering offering complete insurance cover against a random loss, X , where $E[X] = V[X] = 100$ and $\Pr(X > 0) = 1$. The insurer adopts the utility function $u(x) = x - 0.001x^2$ for decision making purposes. Calculate the minimum premium that the insurer would accept for this insurance cover when the insurer's wealth, W , is (A) 100, (B) 200 and (C) 300. 4+6
- d) i) Distinguish between discrete life annuities and continuous life annuities.
- ii) Show that the condition for mutually advantageous policy is $G \geq \mu$, where G is one time premium and μ is expected value of loss random variable. State the assumptions. 4+6

[Internal Assessment: 10]