

U.G. 2nd Semester Examination - 2023

MATHEMATICS

[HONOURS]

Course Code : MATH-H-CC-T-04

(Differential Equations)

Full Marks : 60

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**The symbols and notations have their usual meanings.*

1. Answer any ten questions:

 $2 \times 10 = 20$

- a) Consider $\frac{dy}{dt} = 3y^{\frac{2}{3}}$, $y(0) = 0$. Is the solution of the equation unique? Support your answer.
- b) Deduce the differential equation of all parabolas having their axes parallel to the y-axis.
- c) Solve $\frac{dy}{dx} = \sqrt{y-x}$.
- d) Define Clairaut's form of a differential equation. What is its complete primitive?
- e) If $f(x) = x|x|$, examine if $\frac{d^2 f}{dx^2}$ exists at $x=0$.

[Turn over]

- f) Verify that $(x+y+1)^{-4}$ is an integrating factor of $(2xy - y^2 - y)dx + (2xy - x^2 - x)dy = 0$.
- g) Show that the curve for which the tangent at every point makes a constant angle with the radius vector is an equi-angular spiral.
- h) Solve $(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$.
- i) Consider the equation $t \frac{dy}{dt} - 2y = 0$. Consider the case (a) $y(0) = 0$ (b) $y(0) \neq 0$. For which case the equation will have no solution?
- j) Consider the equation $\frac{dx}{dt} = x^2$. Determine the equilibrium point of the equation. Is the equilibrium point stable?
- k) Find a particular integral of $(D^2 - 4D + 4)y = x^3 e^{2x}$
- l) Solve the equations $\frac{dx}{bcyz} = \frac{dy}{cazx} = \frac{dz}{abxy}$
- m) Determine the characteristics of the equation $u_y(x, y) + ku_x(x, y) = 0$, k is constant.

(2)

n) Solve $p^3x - p^2y$, where $p = \frac{dy}{dx}$.

o) Verify if the following equation is integrable:

$$(y^2 + yz + z^2)dx + (z^2 + zx + x^2)dy + (x^2 + xy + y^2)dz = 0.$$

2. Answer any **four** questions:

$5 \times 4 = 20$

a) Solve the equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 8e^{2x}$$

by the method of undetermined coefficients.

b) Solve $\sin x \frac{d^2y}{dx^2} - \cos x \frac{dy}{dx} + 2y \sin x = 0$.

c) Obtain an analytic solution of the equation

$$\frac{d^2y}{dx^2} + y = 0 \text{ around } x = 0.$$

d) Solve the equation

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$$

by the method of variation of parameter.

e) Solve the partial differential equation

$$px + 3qy = 2(z - x^2q^2)$$

by Charpits method

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(3)

[Turn over]

f) Solve the partial differential equation

$$xz_x - yz_y = \frac{y^2 - x^2}{z}$$

3. Answer any two questions: 10×2=20

a) i) Solve $x \frac{d^2 y}{dx^2} - (2x-1) \frac{dy}{dx} + (x-1)y = 0$.

ii) Verify that x is a solution of the reduced equation of $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = x^2$.

Solve the equation after reducing it to a first order linear equation. 5+5

b) i) Solve $(D^2 + 4)y = \sin 2x$

ii) Solve the system

$$\begin{aligned} (D^2 - 2)x - 3y &= e^{2x} \\ (D^2 + 2)y + x &= 0 \end{aligned} \quad \text{4+6}$$

c) i) Solve the partial differential equation

$$xz_y - yz_x = z$$

with initial condition

$$z(x, 0) = f(x), \quad x \geq 0.$$

ii) Form a partial differential equation by eliminating the arbitrary function F from the relation

$$F(x+y+z, x^2+y^2+z^2) = 0. \quad \text{6+4}$$

