

U.G. 4th Semester Examination - 2022

STATISTICS

[HONOURS]

Course Code : STAT-H-CC-T-9

(Statistical Inference-I and Sampling Distribution)

Full Marks : 50(40+10)

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meaning.*

1. Answer any **five** questions: 2×5=10
- Define a composite hypothesis with an example.
 - When is an estimator said to be unbiased for a function of a parameter? Does such an estimator always exist?
 - Define a t statistic with 5 d.f. What is the mean of this statistic?
 - If X follows $N(0, 1)$, write down the p.d.f. of $|X|$.

- Define Type-I and Type-II errors in connection with hypothesis testing problem. How are the probabilities of these errors controlled?
- How can you carry out large sample test for testing equality of two Poisson parameters?
- Define 'prior' distribution and 'posterior' distribution.
- Define p-value of a test.

2. Answer any **two** questions: 5×2=10
- State Neyman-Fisher factorization theorem. Using this theorem, obtain a sufficient statistic for (θ_1, θ_2) , $\theta_1 < \theta_2$, based on a random sample of size n drawn from a $U(\theta_1, \theta_2)$ population where both the parameters are unknown.
 - If X_1 and X_2 are iid Exponential with mean θ , obtain the distribution of $X_1 + X_2$.
 - Discuss how you would test for the difference of the means of two univariate normal populations with unknown but equal variance on the basis of independent samples from both the populations.
 - Define uniformly minimum variance unbiased (UMVU) estimator. Is it unique? State a

necessary and sufficient condition under which a statistic T is the UMVU estimator of its expectation.

3. Answer any **two** questions: 10×2=20
- a) i) Stating all the necessary assumptions, derive the Rao-Cramer lower bound for the variance of an unbiased estimator of a parametric function. 7
- ii) If Q_1 and Q_3 be the first and third quartiles, respectively, of an F distribution with (n, n) d.f., show that $Q_1 Q_3 = 1$. 3
- b) i) Let X_i ($i = 1, 2, \dots, n$) be iid $N(\mu, \sigma^2)$. Derive the sampling distribution of $\sum (X_i - \bar{X})^2$. 7
- ii) How can you construct a $100(1-\alpha)\%$ confidence interval for the variance of a normal population with unknown mean? 3
- c) i) Explain how an unbiased estimator of a parametric function can be improved with the help of a sufficient statistic. Is it possible to obtain an UMVU estimator by this technique? Justify. 6

- ii) Let $X \sim \text{Bernoulli}(p)$, where p lies in $[1/4, 3/4]$. Show that the MLE of p is $(2X+1)/4$. Also obtain the MSE of this estimator. 4

- d) i) State and prove the additive property of χ^2 distribution. 5
- ii) Write a note on the minimum χ^2 method of estimation. 5

[Internal Assessment: 10]
