

U.G. 4th Semester Examination - 2025**STATISTICS****[MAJOR]****Course Code : STAT-MJ-T-4****(Inference-I and Sampling Distribution)****[NEP-2020]**

Full Marks : 50(40+10)

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meaning.*1: Answer any five questions : $2 \times 5 = 10$

- a) Define a t distribution with $n(>1)$ degrees of freedom. What is the mean of this distribution?
- b) Distinguish between Type-I and Type-II errors in the context of hypothesis testing.
- c) What is contingency table?
- d) Let $(X_1, X_2, \dots, X_n)$ be a random sample from a population with pdf

$$f(x, \theta) = \theta x^{\theta-1}, \quad 0 < x < 1, \theta > 0.$$

Find a sufficient statistic For θ .*[Turn over]*

- e) Distinguish between level of significance and size of a test.
- f) Given a random sample $(X_1, X_2, \dots, X_n)$ from a $N(\mu, \sigma^2)$ distribution, examine unbiasedness of $\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$ for σ^2 .
- g) State Rao-Blackwell theorem.
- h) Distinguish between 'Student's t' and 'Fisher t' distributions with examples.

2. Answer any two questions: 5×2=10

- a) State Fisher-Neyman factorization theorem and prove it in the discrete case.
- b) Describe how you can test for the goodness of fit of a theoretical distribution to a given frequency distribution.
- c) Discuss the p-value approach of the test of significance. How can you test for the mean of a Poisson distribution using this approach?
- d) Discuss how you can construct a $100(1 - \alpha)\%$ confidence interval for the mean of a normal distribution on the basis of a random sample of size n when the variance of the distribution is unknown.

3. Answer any two questions: 10×2=20

- a) i) Let $X \sim \text{Bin}(n, \pi)$, $n \geq 2$. Show that $\left(\frac{x}{n}\right)^2$ is a biased estimator for π^2 . Is it consistent for π^2 ? Justify.
- ii) Let T_1 and T_2 be two unbiased estimators of the same parameter with efficiency e_1 and e_2 respectively. Show that

$$\sqrt{e_1 e_2} - \sqrt{(1 - e_1)(1 - e_2)} \leq \rho \leq \sqrt{e_1 e_2} + \sqrt{(1 - e_1)(1 - e_2)}$$

where ρ is the correlation coefficient between T_1 and T_2 . 5+5

- b) i) Find the moment generating function of Chi-square distribution with n degrees of freedom. Hence or otherwise prove the additive property of Chi-square distribution.
- ii) Let X_1, X_2 be independent random variables with density $f(x) = \exp(-x), 0 < x < \infty$. Show that $Z = X_1/X_2$ has an F-distribution. 5+5

- c) i) Based on two independent random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ populations with all parameters unknown, describe a size α test procedure for testing $H_0: \sigma_1^2 = \sigma_2^2$ against $H_1: \sigma_1^2 \neq \sigma_2^2$.
- ii) Let p be the probability that a coin will fall head in a single toss. In order to test $H_0: p = \frac{1}{2}$ against $H_1: p = \frac{3}{4}$, the coin is tossed 5 times independently and H_0 is rejected if more than 3 heads are obtained. Find the probability of type-I error and power of the test. 5+5
- d) i) Prove that minimum variance unbiased estimator is essentially unique.
- ii) Show, that $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$, based on a random sample of size n from the distribution with density

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} \exp\left(-\frac{x}{\theta}\right), & 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

where $0 < \theta < \infty$ is unknown, is an MVB estimator of θ and has variance $\frac{\theta^2}{n}$. 5+5

[Internal Assessment: 10]