

U.G. 6th Semester Examination - 2021**STATISTICS****[HONOURS]****Discipline Specific Elective (DSE)****Course Code : STAT-H-DSE-3-T****(Stochastic Processes and Queuing Theory)**

Full Marks : 50(40+10)

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meanings.*

1. Answer any **five** questions: $2 \times 5 = 10$
- Define a stationary process.
 - When is a Markov Chain said to be ergodic?
 - Define mean recurrence time.
 - What is an essential state?
 - Write two postulates of a Poisson process.
 - What is the decomposition property of a Poisson process?
 - Write two names of a Queue discipline.
 - Write the limitations of the steady state distribution.

[Turn over]

2. Answer any **two** questions: $5 \times 2 = 10$
- Find the probability generating function of *Binomial* (n, p) distribution.
 - Prove that “Every recurrent communicating class is closed”.
 - Let $\{N(t), t \in [0, \infty)\}$ be a Poisson process with rate λ . Find the probability that there are three arrivals in $(0, 2]$.
 - Describe Little’s Formula.
3. Answer any **two** questions: $10 \times 2 = 20$
- Find the differential equation for $p_n(t)$ in case of pure birth process with rate λ_n . Hence find $p_n(t)$ and p.g.f of Yule Furry process starting with one individual. Identify the distribution. $4 + 4 + 2 = 10$
 - Consider a Markov chain with state space $S = \{0, \dots, N\}$ and transition probabilities $p_{i,i+1} = r, p_{i,i-1} = s$ for $1 \leq i \leq N-1$, where $r + s = 1, 0 < s < 1$; assume $p_{0,1} = 1, p_{N,N-1} = 1$,
 - Draw the graph (transition diagram).
 - Is the Markov chain irreducible? Give reasons.
 - Is it aperiodic? Give reasons.
 - What is the period of the chain?

v) Find the stationary distribution.

$$2+2+2+2+2=10$$

c) State the classical ruin problem. Find an expression for the expected duration of the game which is assumed to be finite.

$$3+7=10$$

d) Consider a queuing system with a single server. The arrival events can be modelled by Poisson distribution with parameter μ . Each customer requires an exponentially distributed service time with mean λ .

i) Draw the state diagram.

ii) Determine p_k using balance equations.

iii) Let $P(z) = \sum_{k=0}^{\infty} z^k p_k$. Calculate $P(z)$ for the system.

$$3+3+4=10$$

[Internal Assessment : 10]
