

U.G. 1st Semester Examination - 2025**PHYSICS****[MAJOR]****Course Code : PHY-MJ-T-01****(Mathematical Physics-I)****[NEP-2020]****Full Marks : 40****Time : $2\frac{1}{2}$ Hours***The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.***GROUP-A****1. Answer any five questions : $2 \times 5 = 10$**

- a) If $A = 2i - 3j + k$ and $A \cdot B = 0$, does it follow that $B = 0$?
- b) Find the slope of the line whose parametric equation is $r = (i - j) + (2i + 3j)t$.
- c) Show that the functions $\cos x$ and $\sin x$ are linearly independent.
- d) Show that if A and B are Hermitian, then AB is not Hermitian unless A and B commute.
- e) If $s = t^u$, find $\partial s / \partial t, \partial s / \partial u$.

[Turn over]

- f) Find the gradient of $w = x^2 y^3 z$ at $(1, 2, -1)$.
- g) Evaluate $\nabla \times \left(\frac{\mathbf{r}}{|\mathbf{r}|} \right)$
- h) Show that $x\delta'(x) = -\delta(x)$.
- i) Three coins are tossed; what is the probability that two are heads and one tails?

GROUP-B

2. Answer any two questions: $5 \times 2 = 10$

- a) i) The rate at which a radioactive substance decays is proportional to the remaining number of atoms. If there are N_0 atoms at $t = 0$, find the number at time t .

ii) Solve $\frac{dy}{dx} - xy = x$, $y = 1$ when $x = 0$.

2+3

- b) $\oiint \mathbf{r} \cdot \mathbf{n} \, ds$ is over the whole surface of the cylinder bounded by $x^2 + y^2 = 1$, $z = 0$ and $z = 3$.

- c) If $u = f(x - ct) + g(x + ct)$, show that
- $$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$$

- d) Solve the differential equation $(D^2 + 5D + 4)y = \cos 2x$.

- e) Evaluate :

i) $\int_0^\pi \sin x \, \delta(x - \frac{\pi}{2}) \, dx$

ii) $\int_{-1}^1 e^{3x} \delta'(x) \, dx$

GROUP-C

Answer any two questions : $10 \times 2 = 20$

3. a) $\iint_A (2x - 3y) \, dx \, dy$, where A is the triangle with vertices $(0, 0)$, $(2, 1)$, $(2, 0)$.

- b) Find the Jacobian of x, y with respect to the polar coordinates r, θ . 6+4

4. a) Find the gradient of $\phi = z \sin y - xz$ at the point $(2, \pi/2, -1)$. Starting at this point, in what direction is ϕ decreasing most rapidly? Find the derivative of ϕ in the direction $2\mathbf{i} + 3\mathbf{j}$.

- b) Evaluate the line integral $\int_C y^2 \, dx + 2x \, dy + dz$, where C connects $(0, 0, 0)$ with $(1, 1, 1)$ along straight lines from $(0, 0, 0)$ to $(1, 0, 0)$ to $(1, 0, 1)$ to $(1, 1, 1)$. 4+6

5. a) Find the eigenvalues and eigenvectors of the following matrix.

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{pmatrix}$$

- b) Find the eigenvalues and eigenvectors of the real symmetric matrix

$$M = \begin{pmatrix} A & H \\ H & B \end{pmatrix}$$

Show that the eigenvalues are real and the eigenvectors are perpendicular. 6+4

6. a) For a simple closed curve C in the plane show by Green's theorem that the area inclosed is

$$A = \frac{1}{2} \oint_C (x dy - y dx).$$

- b) Show that the area inside the ellipse $x = a \cos \theta$, $y = b \sin \theta$, $0 \leq \theta \leq 2\pi$, is $A = \pi ab$. 5+5