

U.G. 4th Semester Examination - 2025

MATHEMATICS

[MAJOR]

Course Code : MATH-MJ-T-4

(Differential Equations)

[NEP-2020]

Full Marks : 60

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**The notations and symbols have their usual meanings.*1. Answer any ten questions: $2 \times 10 = 20$ a) Convert $\frac{dy}{dx} = \frac{xy^2 - y}{x}$ into an exact differential equation.b) Solve $x dx + y dy + \frac{xdy - ydx}{x^2 + y^2} = 0$, given that $y = 1$ when $x = 1$.

c) State Picard's existence and uniqueness theorem.

d) Consider the equation $\frac{dy}{dx} = 3y^{2/3}$, $y(0) = 0$.

Show that it has infinitely many solutions.

[Turn over]

e) Obtain the differential equation of all circles which touch x -axis at the origin.

f) Verify whether $z = 0$ is an ordinary point or a regular singular point of the differential

equation $z^2 \frac{d^2 \omega}{dz^2} + z \frac{d\omega}{dz} + (z^2 - v^2)\omega = 0,$

v is neither zero nor an integer.

g) Obtain the singular solution of the differential

equation $y - px - \frac{1}{p} = 0$, where $p = \frac{dy}{dx}$.

h) Show that the solution $y(x)$ of the initial value problem. $y' = y + \cos x^2$, $y(0) = 0$ exists for $|x| \leq \frac{1}{2}$.

i) Find the Wronskian for the set of functions $\{1, x, x^2\}$ on $[-1, 1]$.

j) Obtain the orthogonal trajectories of the family of hyperbolas $xy = c^2$.

k) Find the partial differential equation by eliminating a and b from the relation $z = (x^2 + a)(y^2 + b)$.

l) Solve by Lagrange's method:

$$x^2 p + y^2 q = z^2$$

m) Given $(p^2 + q^2)x = pz$. Obtain Charpit's subsidiary equations.

n) Determine an integrating factors of the differential equation $(x^3 + y^3)dx - xy^2 dy = 0$.

o) Show that $f(x, y) = xy^2$ satisfies the Lipschitz condition on the rectangle $|x| \leq 1, |y| \leq 1$.

2. Answer any four questions: 5 × 4 = 20

a) Solve by the method of variation of parameters

$$\frac{d^2 y}{dx^2} + y = \operatorname{cosec} x.$$

b) Find the general solution and singular solution of $y = p^2 x^4 - px, \left(p = \frac{dy}{dx}\right)$.

c) Find the general solution of the system of equations

$$\frac{dx}{dt} = 2x + 2y,$$

$$\frac{dy}{dt} = x + 3y.$$

- d) If y_1 and y_2 are two linearly independent solutions of the equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$, where P and Q are continuous functions of x

then show that $y_1 \frac{dy_2}{dx} - y_2 \frac{dy_1}{dx} = ce^{-\int P dx}$,

where c is a non-zero constant.

- e) Show that the equation

$$\sin x \frac{d^2y}{dx^2} - \cos x \frac{dy}{dx} + 2y \sin x = 0$$

is exact and solve it completely.

- f) Find a complete integral of $(p^2 + q^2)y = qz$ by Charpit's method.

3. Answer any two questions: $10 \times 2 = 20$

- a) i) Obtain a series solution of the equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0 \text{ near } x = 0.$$

- ii) Solve by the method of undetermined

$$\text{coefficients, } \frac{d^2y}{dx^2} + 4y = \sin 2x. \quad 5+5$$

- b) i) Reduce the equation

$y^2(y - xp) = x^4 p^2$, $p = \frac{dy}{dx}$, to Clairaut's form by the substitution $x = \frac{1}{u}$, $y = \frac{1}{v}$ and hence solve the equation.

- ii) Solve the following differential equation

$$(x+1)^2 \frac{d^2y}{dx^2} + (x+1) \frac{dy}{dx} = (2x+3)(2x+4). \quad 5+5$$

- c) i) By the use of symbolic operator D , find the solution of the equation

$$\frac{d^4y}{dx^4} - 4\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = e^{2x} \cos x.$$

- ii) Solve :

$$\frac{dx}{x(z^2 - y^2)} = \frac{dy}{y(x^2 - z^2)} = \frac{dz}{z(y^2 - x^2)} \quad 5+5$$

- d) i) Find the characteristics of the equation $pq = z$ and hence determine the integral surface which passes through the parabola $x = 0, y^2 = z$.

- ii) Solve $(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$ by Lagrange's method. $5+5$