

U.G. 6th Semester Examination - 2022

STATISTICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : STAT-H-DSE-T-3

(Stochastic Processes and Queuing Theory)

Full Marks : 50(40+10) Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meanings.*1. Answer any **five** questions: $2 \times 5 = 10$

- Define bivariate probability generating function.
- What is a Markov Chain?
- What do you mean by transient state?
- State Chapman-Kolmogorov equation.
- Write two properties of a Poisson process.
- What is Bulk Queues?
- State Burke's Theorem.
- Write the advantages of the steady state distribution.

2. Answer any **two** questions: $5 \times 2 = 10$

- Find the probability generating function of $X \sim \text{Geometric}(p)$. Hence find $E(X)$.
- Prove that $P^{(n)} = P^n = P.P \dots P$, where P is a transition probability matrix.
- Discuss an application of Poisson Process.
- Explain the inter-relationship between the arrival rate λ and average inter-arrival time. What are the units describing each quantity?

3. Answer any **two** questions: $10 \times 2 = 20$

- Find the solution of a pure death process.
 - Define different steady state measures of performance in a queuing system. $5+5$
- Suppose $i \in S$ of a Markov Chain. Show that i is recurrent if and only if $\sum_{n=0}^{\infty} p_{ii}^{(n)} = \infty$ and transient if and only if $\sum_{n=0}^{\infty} p_{ii}^{(n)} < \infty$.
 - Find the stationary solution for the transition probability matrix $P = \begin{pmatrix} 1-p & p \\ q & 1-q \end{pmatrix}$.

 $5+5$

[Turn over]

- c) i) Show that sum of two independent Poisson process is also a Poisson process.
- ii) Assume a device fails when a cumulative effect of k shocks occurs. If each of the shocks occurs independently according to a Poisson distribution with parameter λ , find the distribution function of the life of the device. 5+5
- d) i) Show that the eigenvalues of a transition probability matrix are in absolute value at most equal to 1.
- ii) Find the distribution of long run waiting time of a customer in M/M/1 system. 5+5

[Internal Assessment : 10]
