

U.G. 2nd Semester Examination - 2022**STATISTICS****[PROGRAMME]****Course Code : STAT(G)CC-T-1B****(Introductory Probability)**

Full Marks : 50(40+10)

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meaning.*

1. Answer any **five** questions: $2 \times 5 = 10$
- Suppose X is a continuous random variable such that $P(X > 0) = 1$. What is the value of $P(X=1)$?
 - The lifetime of an electric bulb is observed until it burns out. Write down the sample space.
 - Write down the p.m.f. of a negative binomial distribution.
 - State the weak law of large numbers.
 - Write down the multiplication law of probability for three events.

- Define moment generating function of a random variable. How does this function generate moments?
- Define convergence in probability of a sequence of random variables.
- Define expectation of a random variable.

2. Answer any **two** questions: $5 \times 2 = 10$

- Define 'pairwise independent' and 'mutually independent' events. Suppose A , B and C are pairwise independent events defined on a sample space such that $P(A) = 1/15$, $P(B) = 2/19$, $P(C) = 1/10$ and $P(A \cap B \cap C) = 1/30$. Find $P(A \cup B \cup C)$.
- For what value of c , the following function would be a p.m.f. of a random variable X ?

$$f(x) = c \cdot (2^x / x!), \quad x = 1, 2, \dots; \text{ and } f(x) = 0, \text{ otherwise.}$$
- Derive the m.g.f. of a Binomial distribution with parameters n and p and hence find the mean of the distribution.
- State DeMoivre-Laplace Central Limit Theorem and mention one application of it.

[Turn over]

3. Answer any **two** questions: $10 \times 2 = 20$

a) i) Define conditional probability. Two cards are drawn at random one by one without replacement from a well-shuffled pack of cards. Find the probability that one is a spade and the other is the queen of hearts.

ii) A shipment of 15 water heaters arrived in a shop was checked and it was found that 4 are with defective thermostats. Two water heaters will be selected at random one after another. Find the probability that (I) both are defective, (II) the second one is defective. $5+5$

b) i) What is Central Limit Theorem? State Lindeberg-Levy Central Limit Theorem.

ii) If $\{X_n, n = 1, 2, \dots\}$ be a sequence of independent random variables with $P\{X_n = n^{1/4}\} = P\{X_n = -n^{1/4}\} = 1/2$, verify whether WLLN holds for the sequence.

$5+5$

c) i) Define distribution function and state its important properties.

ii) Verify whether the following is a distribution function:

$$F(x) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1}x, \quad -\infty < x < \infty. \quad 5+5$$

d) Write notes on:

i) Bayes Theorem

ii) Chebyshev's inequality $5+5$

[Internal Assessment: 10]
