

U.G. 5th Semester Examination - 2020**STATISTICS****[HONOURS]****Course Code : STAH-CC-T-11****(Statistical Inference-II)**

Full Marks : 50(40+10)

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meanings.*

1. Answer any **five** questions: $2 \times 5 = 10$
- Define null hypothesis and alternative hypothesis.
 - What is Type I error in connection with hypothesis testing?
 - Define operating characteristic in connection with sequential testing.
 - When is a test said to be unbiased?
 - What is power of a test?
 - What is the advantage of using an SPRT over a fixed-sample test?
 - Define composite hypothesis with example.
 - Can there be an UMP test which is biased? Justify.

2. Answer any **two** questions: $5 \times 2 = 10$

- State Neyman-Pearson Fundamental Lemma. Show that the test obtained by this Lemma is a function of a sufficient statistic.
- Derive the MP size α test for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1 (< \theta_0)$ on the basis of a random sample of size n from the distribution with p.d.f. $f(x) = (1/\theta) \exp(-x/\theta); x > 0$.
- Describe the likelihood ratio test procedure for testing a composite null hypothesis against a composite alternative. What is the rationale behind this test procedure?
- For the SPRT with stopping bounds A and B , $A > B$, and strength (α, β) show that $A \leq \alpha^{-1}(1 - \beta)$ and $B \geq (1 - \alpha)^{-1}\beta$.

3. Answer any **two** questions: $10 \times 2 = 20$

- Derive the likelihood ratio size α test for testing that the mean of a normal population equals a given value against the two-sided alternatives, based on a random sample of size n from that population, assuming that the population variance is known.

[Turn over]

- b) What is the difference between 'size' and 'level of significance' of a test?

An urn contains 10 marbles of which M are white and $(10 - M)$ are black. To test $H_0: M = 5$ against the alternative $H_1: M = 6$, one draws 3 marbles from the urn without replacement. The null hypothesis is rejected if the sample contains 2 or more white marbles; otherwise it is accepted. Find the size of the test and its power.

- c) What is randomized test? Why is randomization necessary? Suppose X follows binomial distribution with parameters n (known) and p (unknown). Derive the Neyman-Pearson randomized size α test for testing $H_0: p = p_0$ against $H_1: p > p_0$.
- d) Find the SPRT with stopping bounds $B < 1 < A$ of the simple hypothesis $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$ when the underlying distribution is $N(\mu, \theta^2)$ with μ known.

[Internal Assessment: 10]
