

U.G. 6th Semester Examination - 2025**MATHEMATICS****[HONOURS]****Discipline Specific Elective (DSE)****Course Code : MATH-H-DSE-T-04B****(Mechanics-II)****[New Syllabus]****Full Marks : 60****Time : $2\frac{1}{2}$ Hours***The figures in the right-hand margin indicate marks.**The symbols and notations have their usual meanings.*

1. Answer any ten questions: $2 \times 10 = 20$
- a) Define astatic centre for a system of coplanar forces.
 - b) State the principle of virtual work for a rigid body.
 - c) Find the centre of gravity of a semi-circular area of radius a .
 - d) What is meant by stable and unstable equilibrium of a body?
 - e) Show that in a dynamics having six components X, Y, Z, L, M, N the quantity $LX + MY + NZ$ an invariant.
 - f) Explain the terms: 'Wrench' and 'Pitch'.
 - g) Define 'pressure at a point' in a fluid.
 - h) Explain the term 'perfect fluid'.

[Turn Over]

- i) Find the relation between pressure and volume of a fluid if the elasticity is equal to γ times the pressure.
- j) Justify: Depth of C.P. is greater than the depth of C.G.
- k) What do you mean by 'curve of buoyancy'?
- l) Write the condition of equilibrium of a freely floating body in a liquid.
- m) Can fluid be simultaneously 'homogeneous' and 'compressible'? Explain.
- n) What happens to the position of the centre pressure if the plane area is lowered infinitely?
- o) If a homogeneous fluid is in equilibrium under the action of gravity only, then show that the surface of equal pressure is the horizontal planes perpendicular to the lines of force, which are vertical.

2. Answer any four questions: $5 \times 4 = 20$

- a) Prove that a system of coplanar forces not in equilibrium can be reduced to a single force and a single couple.
- b) Show that three coplanar forces P, Q, R acting at the points A, B, C are in astatic equilibrium if they meet at a point on the circumcircle of the triangle ABC and if $P : Q : R = a : b : c$, where a, b, c are the sides of the triangle ABC .
- c) A heavy elastic string whose natural length is $2\pi a$ is placed around a smooth cone whose axis is vertical and whose semi-vertical angle is α . If W be the weight and λ the modulus of the elasticity of the string, prove that it will be in

equilibrium when in the form of a circle whose radius is $a \left(1 + \frac{W}{2\pi\lambda} \cot \alpha \right)$.

- d) OA, OB, OC are three coterminous edges of a cube, and AA', BB', CC' and OO' are diagonals. Along BC', CA', AB' and OO' act forces equal to X, Y, Z , and R , respectively. Show that they are equivalent to a single resultant if $(XY + YZ + ZX)\sqrt{3} + R(X + Y + Z) = 0$.
- e) A solid frustum of a paraboloid of revolution of height h and latus rectum $4a$, rests with its vertex on the vertex of a paraboloid of revolution, whose latus rectum is $4b$. Show that the equilibrium is stable if $h < \frac{3ab}{a+b}$.
- f) Find the equation of the central axis of a given system of forces acting on a rigid body.

3. Answer any two questions: $10 \times 2 = 20$

- a) i) A given volume V of a liquid is acted on by forces $-\frac{\mu x}{a^2}, -\frac{\mu y}{b^2}, -\frac{\mu z}{c^2}$ per unit mass at (x, y, z) parallel to the axes. Find the equation of the free surface.
- ii) Prove with usual notation that the depth of the centre of pressure of a plane area below the horizontal line through the centroid of the area is $\frac{K^2}{h}$. An ellipse of

eccentricity $\frac{1}{4}$ is just immersed with its major axis vertical. Show that the centre of pressure coincides with a focus.

4+(4+2)

- b) i) A cone, whose vertical angle is 2α , has the lowest generator horizontal and is filled with liquid. Prove that the resultant thrust on the curved surface is $\sqrt{1+15\sin^2 \alpha}$ times the weight of the liquid. Also, determine the inclination of the thrust.
- ii) Obtain the necessary conditions for the equilibrium of a mass of liquid under a given system of forces whose components per unit mass at the point (x, y, z) parallel to the coordinate axes are (X, Y, Z) . (4+1)+5
- c) i) Prove that two fluids of different densities at rest under gravity do not mix; their surface of separation is a horizontal plane.
- ii) From a semicircle of radius a whose diameter is on the surface of a liquid, a circle is cut out, whose diameter is the vertical radius of the semicircle. Show that the depth of the centre of pressure of the

remainder is $\frac{9\pi a}{8(16-3\pi)}$. 4+6