

U.G. 2nd Semester Examination - 2020

STATISTICS

[HONOURS]

Course Code : STATH/CC-T-04

(Probability and Probability Distributions-II)

Full Marks : 40

Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer any **five** questions: 2×5=10
 - a) Define a c.d.f. on $\mathbb{R}$. Mention two of its properties.
 - b) When are two random variables said to be independent?
 - c) Define p.g.f. of a discrete random variable. Write down the expression of the p.g.f. of a $\text{Bin}(n, p)$ distribution.
 - d) What does the correlation coefficient ρ between two random variables measure? What can you say about the relationship between the two variables if $\rho = -1$?

[Turn Over]

- e) Mention a situation where Poisson distribution is appropriate to model the data.
 - f) If X and Y are independent and both are distributed as $N(0, 1)$, what is the distribution of the ratio Y/X ? Write down the p.d.f. of this distribution.
 - g) Write down Markov inequality with necessary assumptions, if any.
 - h) If (X, Y) follows bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$ and ρ , what is the marginal distribution of Y and conditional distribution of Y given $X = x$?
2. Answer any **two** questions: 5×2=10
 - a) Define m.g.f. of a probability distribution. Derive the m.g.f. of a Poisson distribution with parameter λ and hence find the mean of this distribution.
 - b) State and prove Chebyshev's inequality.
 - c) Show that all odd order central moments of a normal distribution are zero.
 - d) The random variables X and Y have the continuous joint distribution with p.d.f.
 $f(x, y) = C \exp[-x - y]$ if $0 < x < y < \infty$; and 0 otherwise.
 Find the marginal distributions of X and Y .

3. Answer any **two** questions: $10 \times 2 = 20$

a) i) Verify whether the function

$F(x) = 0$ if $x \leq 1$ and $1-x$ if $x > 1$
represents a c.d.f. 4

ii) For any integer-valued random variable X , show that

$\sum_{n=0}^{\infty} s^n P\{X \leq n\} = (1-s)^{-1} P(s)$, where
 P is the p.g.f. of X . 6

b) i) For any bivariate distribution of (X, Y) ,
show that

$\text{Var}(Y) = \text{Var}[E(Y|X)] + E[\text{Var}(Y|X)]$.
5

ii) Write a note on lognormal distribution
and its uses. 5

c) i) Suppose (X, Y) follows bivariate normal
distribution with $E(X) = E(Y) = 0$,
 $\text{Var}(X) = \text{Var}(Y) = 1$ and $\text{cov}(X, Y) = \rho$.
Show that $Z = Y/X$ follows Cauchy
distribution. Identify the parameters of
this distribution. 5

ii) Show that $\beta_2 \geq \beta_1 + 1$. 5

d) Write down the p.d.f. of an exponential
distribution with mean λ . State and prove the
lack of memory property of this distribution.
Mention some of the uses of this distribution.

$2+5+3=10$
