

U.G. 2nd Semester Examination - 2020**STATISTICS****[PROGRAMME]****Course Code : STAT(G)CC-T-1B****(Introductory Probability)**

Full Marks : 40

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.*

1. Answer any **five** questions: $2 \times 5 = 10$
- Define 'random experiment' with an example.
 - A coin and a die are flipped simultaneously. Write down the sample space.
 - Give one example each for a discrete random variable and a continuous random variable.
 - State the weak law of large numbers.
 - Write down the p.m.f. of a Poisson distribution and expressions of its mean and variance.
 - Define moment generating function of a random variable. How does this function generate moments?

[Turn over]

- Give the statistical definition of probability.
- What is the probability of getting a sum of 9 in throwing two fair dice simultaneously?

2. Answer any **two** questions: $5 \times 2 = 10$

- Give the classical definition of probability. What are the limitations of this definition? Ten cards are drawn at random from a full pack of well-shuffled cards. What is the probability of finding (i) at least one ace (ii) exactly one ace?
- Obtain the value of c for which the following function $f(x)$ would be a p.m.f. of a discrete random variable X :

$$f(x) = c(x-1), x = 1, 2, \dots, 10$$

$$= 0, \text{ elsewhere.}$$

Also obtain the expectation of X .

- Write down the p.m.f. of a binomial distribution. Derive the mean of this distribution.
- Verify whether weak law of large numbers holds for the following sequence of independent random variables $\{X_n, n = 1, 2, \dots\}$:

$$P\{X_n = 2^n\} = P\{X_n = -2^n\} = \frac{1}{2}.$$

3. Answer any **two** questions: $10 \times 2 = 20$

- a) i) State and prove Boole's inequality. 5
ii) A_1 and A_2 are two events related to a random experiment. Given $P(A_1) = \frac{1}{2}$, $P(A_2) = \frac{1}{3}$ and $P(A_1 \cap A_2) = \frac{1}{4}$. Determine $P(A_1^c \cup A_2^c)$ and $P(A_1^c \cap A_2)$.

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- b) State and prove Bayes' Theorem.

The probability that a doctor will diagnose a certain disease correctly is 0.6. The probability that a patient will die by his treatment after correct diagnosis is 0.4. The probability that the patient will die after wrong diagnosis is 0.7. Suppose that a patient having the disease was treated by the doctor and died. What is the probability that the disease was diagnosed correctly?

- c) What is Central Limit Theorem? State Lindeberg-Levy Central Limit Theorem. Why is this CLT said to give a more precise statement about the sample mean than the weak law of large numbers?

- d) Establish Chebyshev's inequality. Using this inequality give an upper bound to the probability that the absolute difference of a random variable from its mean will be greater than thrice the standard deviation.
