

U.G. 4th Semester Examination - 2025**MATHEMATICS****[MINOR]****Course Code : MATH-MN-T-2****(Calculus and Differential Equations)****[NEP-2020]**

Full Marks : 40

Time : 2 Hours

*The figures in the right-hand margin indicate marks.**The notations and symbols have their usual meanings.*

1. Answer any five questions: 2×5=10

a) Give an example to show that the sum of two discontinuous functions can result a continuous function.

b) Find the value of α that makes the function

$$f(x) = \begin{cases} \frac{2 \tan x}{x}, & \text{if } x \neq 0 \\ \alpha, & \text{if } x = 0 \end{cases}$$

continuous at $x = 0$.

c) Examine whether the function

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

is differentiable at $x = 0$.

d) If $y = 2 \cos x (\sin x - \cos x)$, show that $(y_{10})_0 = 2^{10}$.

e) Prove that a parabola has no asymptote.

[Turn over]

f) Show that for no real values of k the equation $x^2 - 3x + k = 0$ has two distinct roots in $[0, 1]$.

g) Evaluate $\lim_{x \rightarrow \infty} \left[x - \sqrt[3]{(x-a)(x-b)(x-c)} \right]$, where $a, b, c > 0$.

h) Let $f(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$

Prove that $\frac{\partial^2 f}{\partial x \partial y}(0, 0) \neq \frac{\partial^2 f}{\partial y \partial x}(0, 0)$.

2. Answer any two questions: $5 \times 2 = 10$

a) State Leibnitz's theorem for the n^{th} derivative of product of two functions.

If $y = \frac{1}{\sqrt{1+2x}}$, prove that $(1+2x)y_{n+1} +$

$(2n+1)y_n = 0$, where n is a positive integer.

Hence evaluate $(y_n)_0$. $1+2+2$

b) Expand $\log_e(1+x)$ in a power series of x when $x > -1$. 5

c) State Euler's theorem for a homogeneous function of two variables.

Verify that $\cos V = \frac{x+y}{\sqrt{x} + \sqrt{y}}$ is a homogeneous

function of degree $\frac{1}{2}$ in x and y . Hence prove

that $x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} + \frac{1}{2} \cot V = 0$. $1+2+2$

d) If ρ_1, ρ_2 be the radii of curvature of a focal chord of a parabola having semi-latus rectum l , then

prove that $\rho_1^{-\frac{2}{3}} + \rho_2^{-\frac{2}{3}} = l^{-\frac{2}{3}}$. 5

3. Answer any two questions: $10 \times 2 = 20$

a) i) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1 + \tan x}{1 + \sin x} \right)^{\frac{1}{\sin x}}$.

ii) If $H = f(y-z, z-x, x-y)$, prove that

$$\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0.$$

iii) Find the existence of optima of

$$f(x, y) = x^2 + y^2 + (x+y+1)^2. \quad 4+3+3$$

b) i) Find the rectilinear asymptotes of the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$.

ii) Find the radius of curvature of the curve $y = xe^{-x}$ at the point where y is maximum. $5+5$

c) i) If $I_n = \int_0^{\pi/2} x^n \sin x \, dx$ ($n \geq 1$), then show

$$\text{that } I_n + n(n-1)I_{n-2} = n \left(\frac{\pi}{2} \right)^{n-1}.$$

Hence show that

$$\int_0^{\pi/2} x^4 \sin x \, dx = \frac{\pi^3}{2} - 12\pi + 24.$$

ii) Find the general solution of the differential

equation: $x \cos\left(\frac{y}{x}\right)(ydx + xdy) = y \sin\left(\frac{y}{x}\right)$

$(xdy - ydx).$ 5+5

d) i) Reduce the equation $x^2(y - px) = p^2y$ to Clairaut's form by making the substitution $x^2 = u$ and $y^2 = v$, and then find its general

solution $\left(p \equiv \frac{dy}{dx}\right).$

ii) Solve the differential equation:

$\frac{d^2y}{dx^2} + a^2y = \sec ax$, by the method of

variation of parameters. 5+5