

U.G. 4th Semester Examination - 2021**STATISTICS****[HONOURS]****Course Code : STAT-H-CC-T-9****[Statistical Inference-I and Sampling Distribution]**

Full Marks : 20

Time : 1 Hour

*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.*

1. Answer any **five** questions: 1×5=5
- What is the problem of estimation of a parameter? Give an example.
 - When is an estimator said to be consistent for a parameter? State sufficient conditions for an estimator to be consistent.
 - Define a chi-square statistic. State the additive property of chi-square distribution.
 - If X follows t distribution with n d.f., what is the distribution of $Y = -X$? Justify your answer.

- Define Type I and Type II errors in connection with hypothesis testing problem. How are the probabilities of these errors controlled?
- How can you carry out large sample test for testing equality of two binomial proportions?
- What is posterior distribution? How is Bayes estimator of a parameter found under squared error loss?
- Define moment generating function of a random variable, How does it generate moments?

2. Answer any **one** question: 5×1=5
- $X_1, X_2, \dots, X_n$ are iid observations from a population with p.d.f.

$$f(x|\theta) = \theta \exp(-\theta x), x > 0; \theta > 0.$$
 Obtain the maximum likelihood estimator of θ . Is it unbiased?
 - If X_1 and X_2 are iid $U(0, a)$, find the p.d.f. of $X_1 + X_2$.
 - Suppose T_1 and T_2 are unbiased estimators of θ with known variances σ_1^2, σ_2^2 and correlation coefficient ρ . Obtain the best linear unbiased estimator of θ based on T_1 and T_2 .

3. Answer any **one** question: $10 \times 1 = 10$
- a) i) State Fisher-Neyman Factorization theorem and prove it for the discrete case. 7
- ii) Using Factorization theorem, derive a sufficient statistic for the unknown location parameter θ of a Cauchy population with scale parameter unity based a random sample of size n . Is it complete? 3
- b) i) Let X_i ($i = 1, 2, \dots, n$) be independently distributed as normal variables with means μ_i and common variance σ^2 . Show that $\bar{X}$ and $\sum (X_i - \bar{X})^2$ are independently distributed. 6
- ii) Suppose n independent observations are available from a Poisson distribution with mean λ . Suggest a large sample test for testing the null hypothesis $H_0 : \lambda = \lambda_0$ against the alternative $H_1 : \lambda = \lambda_0$. 4
- c) i) State and prove Rao-Blackwell theorem. Give the implication of the theorem in point estimation. 6

- ii) Describe minimum χ^2 method of estimation. State its demerits. 4

(Internal Assessment : 5)
